VELOCITY PROFILING IN HYDROTRANSPORT

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Abstract: An algorithm for the calculation of the rate of flow from a five sensor velocity profiler was developed. The algorithm produces flow rate values that are to within a few percent of available reference flow rate values for polydisperse sand up to the point where a stationary sand bed is laid down. An operational definition of sanding slurry flows is formulated. Based on this definition a metric, a ζ value, is developed as the ratio of the absolute value of the velocity gradient (shear rate) near the bottom of the pipe to the absolute value of the velocity gradient (shear rate) near the top of the pipe. This ζ value appears to be a useful early warning quantity in polydisperse sand slurry flow but completely fails to indicate sanding behavior in monodisperse sand slurry flow. Field experience of the five sensor array velocity profiler in a big bore hydro-transport line is presented showing the behavior of the meter under conditions representative of the flow of sand and water mixtures in practical conditions.

Key words: velocity profile, polydisperse, monodisperse, shear rate, stratification

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1 INTRODUCTION

In hydro-transport, solids; often sand of variable composition and coarseness, are being transported in a big bore pipeline using water as the carrier fluid. The pipeline may be short or long, up to 35 km, and it will invariably be exposed to some terrain undulation causing upward and downward flow with the changes in inclination. The pull of gravity has a strong influence on the characteristics of the flow of the mixture of carrier fluid and sand slurry. In horizontal pipe lines it may lead to heavily stratified flow, in vertical upward flow the distribution of the carried solids tends to be more homogeneous. In extreme cases of low flow velocity and high solids concentration a solids bed may be deposited on the bottom of the pipe line in horizontal flow. Such a bed may be experiencing stick/slip flow or it may be completely stationary.

The prediction of the pressure drop in such hydro-transport lines is by no means a small task. A large number of models exist in the literature and a small subset thereof is used in practice. Under predicting the pressure loss may lead to completely sanding out the line where a deposited bed grows to block the flow. Over predicting the pressure loss is undesirable from an economic point of view as it will result in over sized pumping requirements and fuel consumption. A measurement device which will reliably indicate the degree of stratification as well as measure the true volumetric rate of flow, irrespective of the stratification, may help in two different ways. Such a device could be used to develop better, more accurate, pressure drop models over a larger range of conditions or at least over a range of conditions described by in-situ conditions. Moreover such a device could be used to actively control the line speed to maintain a certain level of acceptable stratification independent of solids density, make up (silt, sand, clay, gravel) and independent of solids coarseness.

Based on sonar array velocity measurement [1] it is possible to measure multiple velocities at a number of locations using sensing material mounted on the outside pipe wall. Such velocities indicate the fluid flow velocity at various heights in a vertical cross section of the pipe. Experience has shown that these raw measured velocities do show heavily skewed profiles in stratified flow and flat profiles in single phase water flow. Since traditional flow meter calibration cannot be applied, a different way of evaluation of the accuracy of such skewed profiles is necessary. Additionally it would be of interest to develop a metric indicating the degree of stratification based on the skewness of the measured velocity profile.

Passive sonar based mixture velocity measurement [2], whilst widely accepted in many industries including hard rock mining, is an emerging technology in the dredging world. Applications in dredging include dredging mixture velocity measurement on both hopper and cutter dredgers as well as Gas Void Fraction (GVF) measurement for vacuum degasser control.

Multiple velocity measurements along the perimeter of a pipe are possible. A 5 point velocity profile at the top, upper, mid, lower and bottom sections is routinely measured. Using the no-slip condition at the top and bottom, 7 points are available, positioned at the nodes of a 7^{th} order Chebyshev polynomial. Interpolation and assuming a fully developed turbulent profile in the horizontal direction gives a 2-D velocity profile. Numerical integration thereof gives the average flow velocity. Numerical differentiation of the (approximate, interpolated) vertical velocity profile gives the rate of shear.

The average velocity directly gives the rate of volumetric flow. The rate of shear provides insight into the rheology of the mixture because the wall rate of shear is now known. If differential pressure were available in order to infer the wall shear stress then the ratio of (wall) shear stress to (wall) shear rate gives (local) viscosity.

In heavily stratified flows, including flows with a sliding bed, the ratio of the shear rate at the top of the pipe to the shear rate at the bottom yields a ζ parameter which has proven to be a good metric for the degree of stratification. This ζ may serve as a predictor of sanding or laying down a stationary sand bed.

Results obtained in a flowloop as well as field results from various hydro-transport applications will be shown. Operating a hydro-transport line as close as is possible to near sanding conditions will lead to quantifiable efficiency increases and contribute to more environmentally friendly operations by saving fuel.

Objectives: In October 2007 experiments were carried out at the Saskatchewan Research Council (SRC) using a variety of sonar based flow meters including a five sensor array velocity profiler and a passive sonar-based flow meter which provides the average flow velocity of the slurry mixture.

The experiments included tests of flow meter performance in sanding conditions of flow with both monodisperse sand as well as polydisperse sand. The objectives of this work can be summarized as:

1. Develop an algorithm to calculate the rate of flow from the five measured velocities returned as raw data from the five sensor array velocity profiler.

2. Compare the rate of flow calculated with the available reference flow rate and with the flow rate as reported by a segmented band, passive sonar array flow meter.

3. Develop an operational definition of sanding conditions using measurable quantities.
4. Develop a metric for sanding which allows for early warning of the occurrence of sanding conditions.
5. Determine a set of limits for sanding conditions on which a tristate alarm (green = safe, orange = warning, red = alarm) can be based.

With only five measured velocities at our disposal it is not easy to find a way to directly compute a volumetric flow rate. The five measured velocities are therefore used to find an approximation to the full two dimensional velocity profile. This does require a number of assumptions.

1. It is assumed that the horizontal velocity profile can be described by a fully developed turbulent velocity profile at any vertical location in the pipe.
2. It is assumed that there is no slip at the pipe wall as is customary in fluid mechanics.

The above two assumptions rule out a computation in the case of a sliding bed of solids deposited on the bottom as both of the above assumptions are violated in such a case. The assumptions allow for asymmetry in the vertical direction, e.g. as caused by stratifcation.

2 SENSOR PLACEMENT

The placement of the five sensors on the outside wall of the pipe is such that the projection thereof to the vertical cross section (scaled such that the pipe bottom is at -1 and the top of the pipe is at +1) coincides approximately with the nodes of a Chebyshev polynomial of degree seven. Orthogonal Chebyshev polynomials are given by:

$$T_n(y) = \cos(n \arccos(y)) \tag{1}$$

The 5^{th} order and 7^{th} order Chebyshev polynomials are depicted in 1 in the right hand panel. The left hand panel provides a cross section of a pipe (scaled to vertical domain of $[-1, +1]$). This cross section shows the location of the nodes of a 7^{th} order Chebyshev polynomial on the left hand side and the approximate position of the sensor array sense points on the right hand side. For comparison the nodes of a 5^{th} order polynomial are also shown. The colored circle arcs show the approximate location and extent of the sensing material on the outside of the pipe wall. The colors are chosen to match the curve colors in the graph tracks giving the measurement results. The position of the measurand along the pipe perimeter is by number and by color (1 top; 2 upper; 3 mid; 4 lower; 5 bottom) in Figure 1. Note that in this cross section the array of eight sensors aligned co-axially is not shown. Sonar array processing derives a velocity from a phase shift induced by hoop strains along the array. A sonar array is sensitive to coherent power propagating co-axially through the array [1]. Therefore the actual extent of the sensors is much less important than one may think.
The calculation of a Chebyshev approximation requires that the function to be approximated (the vertical velocity profile in our case) is evaluated (measured) at the nodes (zeros) of an N^{th} order Chebyshev polynomial [3]. There are N such nodes for an N^{th} order polynomial. These nodes y_k are given by:

$$y_k = \cos\left(\frac{\pi(k - 1/2)}{N}\right) \tag{2}$$

Graphically this can be represented as in Figure 1. A tabular summary is given in Table 1. The node location corresponds to the y co-ordinate of the zeros of the Chebyshev polynomial of the respective order.
An optimized array of five sensors would be possible by adjusting the sensor locations to match the position of the inner 5 nodes of a 7^{th} order Chebyshev polynomial. Optimization requires the fit of the 45° sensor to 64° and the 135° sensor to 116°.
The actual extent of the sensor as a fraction of the perimeter is, surprisingly, not critical. It is to be understood that the velocity is derived from an array of sensors by a small phase difference between the eight sensors (in the axial direction) comprising an array.

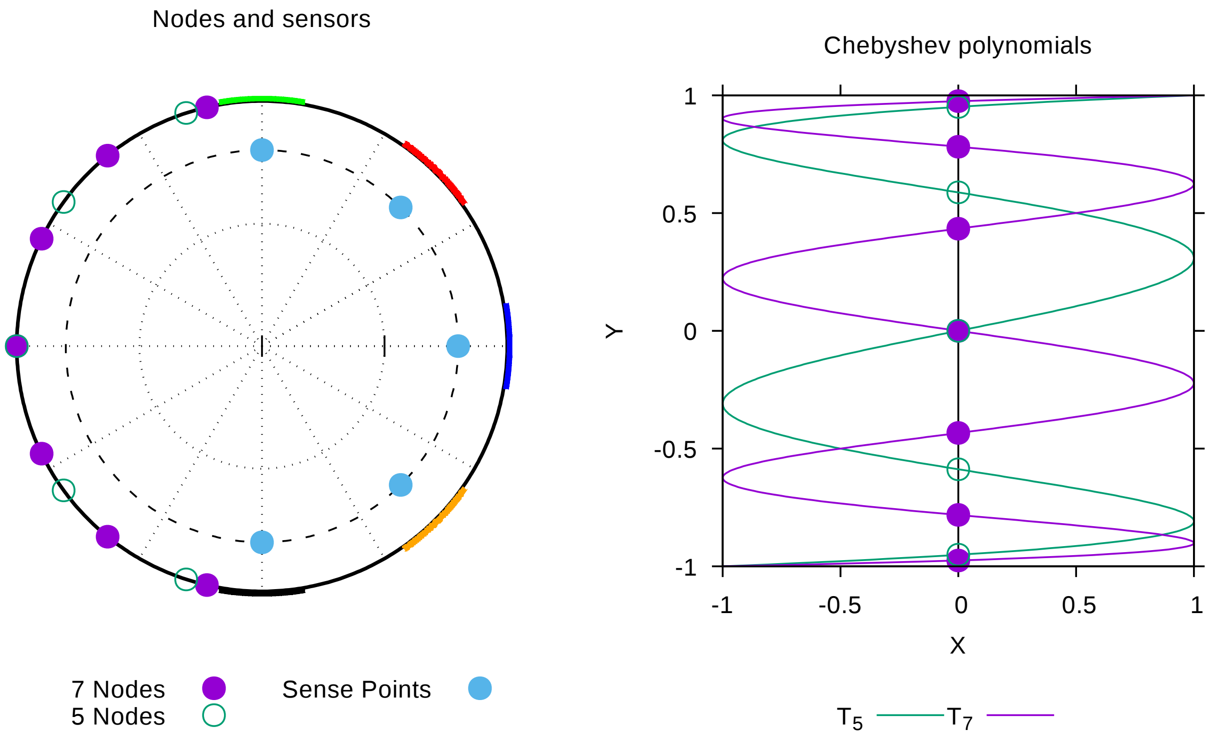

Figure 1: Sensor placement

2.1 Velocity profile construction

Given that the sensor locations are close to the nodes of a 7^{th} order Chebyshev polynomial, the coefficients c_j of this polynomial can be calculated as:

$$c_j = \frac{2}{N} \sum_{k=1}^{N} V_k \cos\left(\frac{\pi j (k - 1/2)}{N}\right) \tag{3}$$

where $N = 7$ and V_k is the measured (or computed) velocity numbered from the top down to the bottom. Note that V_2 is the top sensor, V_3 is the upper sensor, V_4 is the middle sensor, V_5 is the lower sensor and V_6 is the bottom sensor. V_1 and V_7 are computed velocities which are initially set to zero. These are used to enforce the no slip condition at the wall.

Table 1: Sensor layout and nodes of Chebyshev polynomials

Chebyshev order	Node location	Angle equivalent	Sensor location	Angle equivalent	Velocity number
7	+0.975	13°			V_1 computed
	+0.782	39°	+0.860	0°	V_2 measured
	+0.434	64°	+0.606	45°	V_3 measured
	0.000	90°	0.000	90°	V_4 measured
	-0.434	116°	-0.608	135°	V_5 measured
	-0.782	141°	-0.860	180°	V_6 measured
	-0.975	167°			V_7 computed
5	+0.951	18°	+0.860	0°	
	+0.588	54°	+0.606	45°	
	0.000	90°	0.000	90°	
	-0.581	126°	-0.608	135°	
	-0.951	162°	-0.860	180°	

An approximate vertical velocity profile can now be computed as per:

$$\nu(y) = \sum_{k=0}^{N-1} c_k T_k(y) - \frac{1}{2} c_0 \tag{4}$$

This approximation is exact at the nodes of the 7^{th} order polynomial.

2.1.1 No slip condition at the wall

The velocity at the bottom or top of the pipe (which must be zero according to the no slip condition) can be computed as:

$$\nu(\pm 1) = \sum_{k=0}^{N-1} c_k T_k(\pm 1) - \frac{1}{2} c_0 = \frac{2}{N} \sum_{k=0}^{N-1} \left(T_k(\pm 1) \sum_{j=1}^{N} V_k \cos\left(\frac{\pi k (j - 1/2)}{N}\right)\right) - \frac{1}{2} c_0 = 0 \tag{5}$$

This velocity $\nu(\pm 1)$ is forced to zero (within a specified tolerance of, say, 0.001 fps) by iteratively updating the velocity V_1 and V_7, whilst keeping all other velocities at their measured values. As a starting values $V_1 = 0$ and $V_7 = 0$ are used. The iterative procedure which is employed is a numerical root finder as given by Brent [3].

2.2 Flow rate computation

A non-dimensional velocity profile in two dimensions is given by the equation below which basically is the profile of a fully developed turbulent profile at height y in the pipe (y running from -1 to +1) with a maximum velocity given by v(y) the Chebyshev approximated vertical velocity profile.

$$u(x, y) = \nu(y) \left(1 - \frac{x}{\sqrt{1 - y^2}}\right)^{\frac{1}{n}} \tag{6}$$

In this equation n is a shape factor of the profile. Higher values of n lead to more flat topped profiles. In this work a value of $n = 7$ is chosen. Note that this velocity profile is a two dimensional profile written in terms of Cartesian co-ordinates x and y. Because of the circular geometry the x co-ordinate's domain depends on the value of the y co-ordinate. Thus in horizontal planes the velocity at the wall is always zero, irrespective of the vertical location of such a plane.

2.2.1 Analytical and numerical integration

This velocity profile can be integrated to find the rate of flow as follows:

$$q = 2 \int_{-1}^{+1} \nu(y) \int_{0}^{\sqrt{1-y^2}} \left(1 - \frac{x}{\sqrt{1 - y^2}}\right)^{\frac{1}{n}} dx dy \tag{7}$$

Observe that the inner integral over the horizontal co-ordinate x runs from 0 to $\sqrt{1 - y^2}$. This is necessary to take into account the varying width of a velocity profile in a horizontal plane with varying vertical location in the pipe. The inner integral can be done analytically, which results in:

$$q = 2 \frac{n}{n+1} \int_{-1}^{+1} \nu(y) \sqrt{1 - y^2}\, dy \tag{8}$$

where it is apparent that the rate of flow equals the sum over slices with length $2\sqrt{1 - y^2} dy$ multiplied with the average velocity $\frac{n}{n+1} \nu(y)$. This integral is computed by numerical quadrature. The routine chosen is a Romberg scheme driving an extended trapezoidal rule [3].

Last but not least, the rate of flow is dimensionalized by multiplying with the square of the inside pipe radius. Allowance is made for a factor C for unit conversion purposes. The end result is an equation which expresses the rate of flow in engineering units such as $m^3 s^{-1}$, provided that the velocity is expressed in ms^{-1}.

$$Q = CR^2 q \tag{9}$$

A variety of conversion factors C will be necessary if the rate of flow is desired in other units of flow rate. The factor C only depends on the desired units of flow rate. Normal flow measurement using passive sonar based array sensors require calibration. The offset, i.e. the relative deviation of the measured velocity from the true average volumetric velocity is linear in the inverse square root of the Reynolds number. No such calibration is used in this work.

2.3 Sanding metric development

For the purposes of this work an operational definition of a sanding flow condition is required that does not involve any reference to (local) density. Instead a definition is required which is formulated entirely in terms of velocities or quantities that can be directly derived from velocity and length such as a velocity gradient (a shear rate). The following definition of sanding is therefore proposed:

Definition 1. A sanding condition is present if and only if the shear stress near the bottom of the pipe is lower than the shear stress near the top of the pipe.

This definition is difficult to implement. The shear stress τ in a one dimensional flow at the vertical location y equals:

$$\tau = -\eta \frac{\partial v}{\partial y}\Big|_{y} \tag{10}$$

The problem here is that in non-Newtonian flows (such as slurry flows) the value of the dynamic viscosity η is not well known. Moreover the dynamic viscosity will be dependent on the (local) concentration of fines such that there is a possibility that the dynamic viscosity at the top of the pipe differs from the dynamic viscosity at the bottom of the pipe.

2.3.1 Shear rate ratio

Therefore a more practical metric for the presence of a sanding condition can only be given in terms of the ratio of the shear rates near the bottom and near the top of the pipe.

$$\zeta = \frac{\frac{\partial v}{\partial y}\Big|_{y=-1}}{\frac{\partial v}{\partial y}\Big|_{y=+1}} \tag{11}$$

This ζ ratio is easily calculated. From the Chebyshev approximated velocity profile a derivative is readily available [3]. The advantage of the above metric is that it references the conditions near the top of the pipe. Hence, any variation in the dynamic viscosity of the carrier liquid of the slurry (water most likely) will not influence the metric.

Table 2: Proposed sanding metric limits

Safe	Warning	Alarm
$\zeta \geqslant 1$	$0.5 < \zeta < 1$	$\zeta \leqslant 0.5$

A proposed setting for a tristate warning system is given in Table 2. In most cases it will be found that these settings need to be customized to the local environment for optimal performance.

3 LABORATORY EVALUATION

In October 2007 a series of experiments was carried out at the Saskatchewan Research Council (SRC) using a variety of sonar based flow meters including a five sensor array velocity profiler and a segmented band flow meter. An overview of the flow loop is given in Figure 2. Experiments reported upon in this work were all performed with the meter designated as *5 sector array*. Results obtained from other instruments is not reported on in this work.
The objectives of this work can be summarized as:

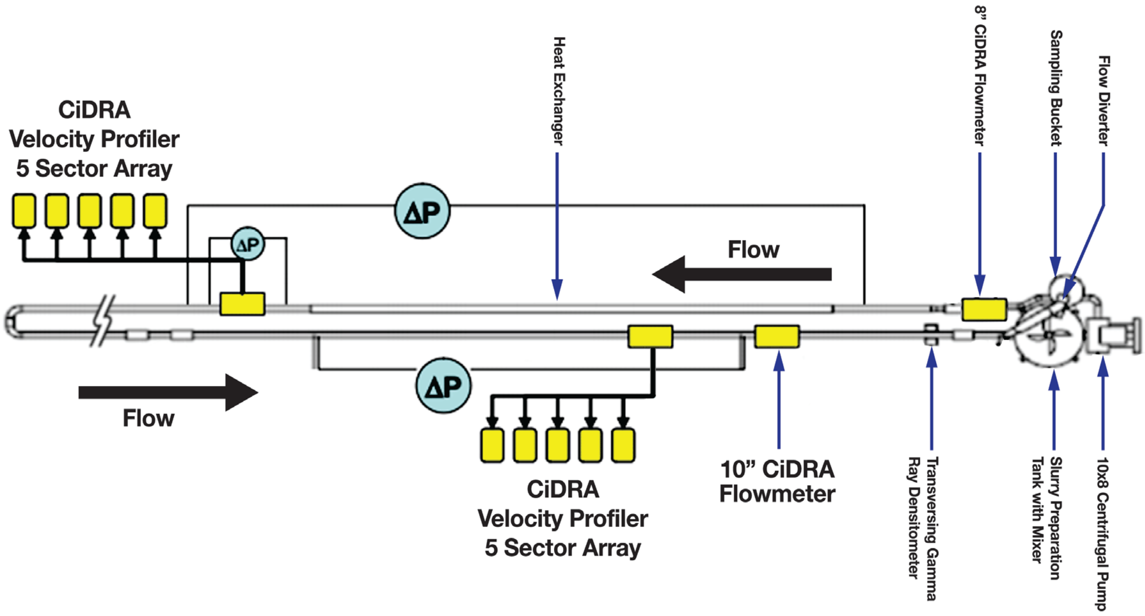

Figure 2: Flowloop layout

1. Develop an algorithm to calculate the rate of flow from the five measured velocities returned as raw data from the five sensor array velocity profiler.
2. Compare the rate of flow calculated with the available reference flow rate and with the flow rate as reported by a segmented band flow meter.
3. Develop an operational definition of sanding conditions using measurable quantities.
4. Develop a metric for sanding which allows for early warning of the occurrence of sanding conditions.
5. Determine a set of limits for sanding conditions on which a tristate alarm (green=safe, yellow=warning, red=alarm) can be based.

The experiments included tests of flow meter performance in sanding conditions of flow with both monodisperse sand as well as polydisperse sand. Monodisperse sand is sand which has been classified by hydro-cyclones into a fine fraction normally used for flotation. The dispersion of this sand in terms of the width of the particle size distribution is small. The median diameter was found to be 169μm. A depositional velocity of the flow can be predicted below which a sand bed will rapidly form. The model used to this extent is the in-house SRC model. Under such conditions the velocity profile is not expected to show much stratification. Polydisperse sand has a wider particle size distribution. The median diameter was determined to be 324μm. Because of the presence of very coarse as well as very fine particles it was not possible to predict a depositional velocity. It is expected that the velocity profile will show varying degrees of stratification dependent on the local density and the instantaneous line speed.

3.0.1 Figure layout

Because of the amount of data and the complexity thereof a special graphical layout is used for reporting of results. The data is shown in tracks with a time based ruler progressing downwards. To the left of the time based ruler measured data is given in 3 (velocities, qualities and sound pressure levels) or 4 (velocities, qualities, sound pressure levels and temperatures) tracks. To the right of the ruler computed results (velocity profile, average flow velocity, shear rate profile, ζ ratio). For the flow loop data a density track is added ar the far right. The unit of the measurand is given at the top from a minimum value at the left to a maximum value at the right for each of the data tracks. The profiles are given with the left of the track representing the bottom of the pipe and the right of the track the top of the pipe. The computed value is represented by a color scale. The position of the measurand along the pipe perimeter is by number and by color (1 top; 2 upper; 3 mid; 4 lower; 5 bottom) in Figures 3, 4, 5 and 6.

Velocity track: Five velocity traces are given, one for each of the sensor arrays. The unit of measurement is $ft.s^{-1}$ (feet per second).

Quality track: Five quality traces are plotted, one for each of the sensor arrays. Quality is a dimensionless value in the range $[0, 1]$. The higher the quality, the better the measured velocity was determined. Below a quality value of 0.2 the corresponding velocity measurement is usually rejected.

Sound Pressure Level (SPL) track: Five SPL traces are drawn, one for each of the sensor arrays. The unit of measurement is dB relative to $20\mu Pa$. SPL can be a helpful quantity in determining the process status. A process that is running typically has higher SPL values than processes not running.

Temperature track: Five sensor array, outside of pipe, temperature traces are graphed if available. Temperature is sometimes very useful in a qualitative way as the dominant mode of heat transport by convection changes when a sand bed is deposited. The unit of measurement is $^\circ C$.

Velocity profile track: A computed velocity profile is given as a false color image. At the top the color scale maps to velocity which is given in $ft.s^{-1}$. The black over plotted line is the position in the pipe where the velocity attains its maximum value. The left hand side of the track corresponds to pipe bottom. The right hand side of the track corresponds to the top. The diameter is scaled from -1 to $+1$ in order to cater for different pipe sizes. This scaling is also required for Chebyshev approximation.

Average velocity track: Double integration of the velocity profile as per equation 7 and equation 8 results in the average flow velocity (black) which is given in this track in units of $ft.s^{-1}$. The blue curve is the average velocity as derived from the reference flow meter. The red curve is relative deviation between these two on a scale of -10% to $+10\%$.

Shear rate track: Differentiation of the velocity profile results in the local shear rate in the vertical plane. Since diameter has been scaled to be within -1 to $+1$, the unit of shear rate here is somewhat awkward: ft.

ζ **ratio track:** As per equation 11 the ratio of shear rates at the bottom to that at the top is drawn on a logarithmic scale in this track. The various values of the proposed tristate alarm in table 2 are given as filled curves.

Density track: The bottom of pipe mounted radiometric density in units of kgm^{-3} is given here as a reference. It is only available in the case of the flow loop data. The range is $1250kgm^{-3}$ to $2250kgm^{-3}$.

3.1 Polydisperse sand

Observe how the ζ ratio drops to low values, well below the safe or warning limits almost immediately at 15:22h. The value continues to decrease until the very end at 16:30h when a bed is laid down. x

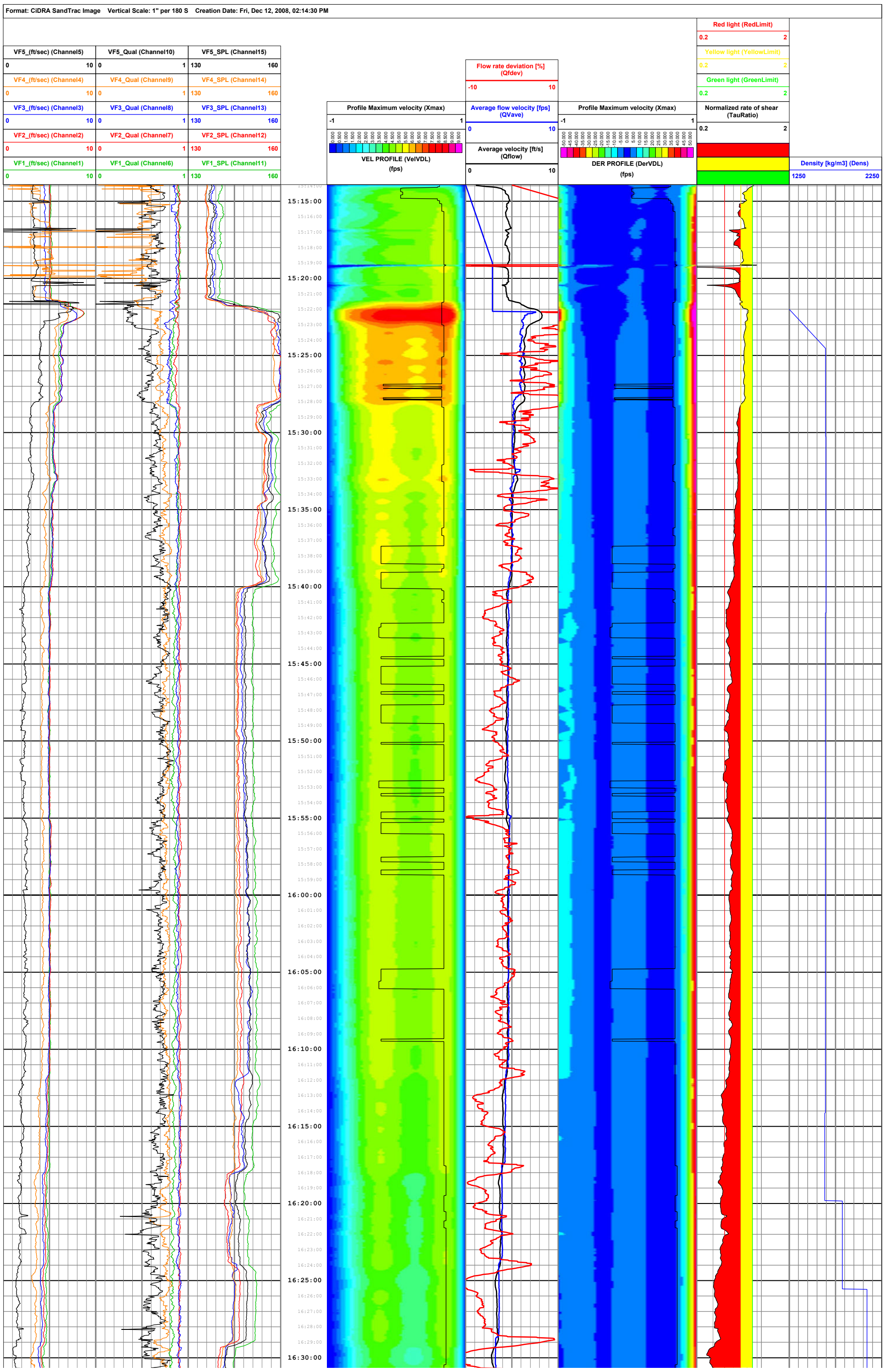

Figure 3: Poly-dispersed sand

3.2 Monodisperse sand

Contrary to the case of polydisperse sand it is shown here that virtually no stratification is observed. Neither the raw velocity data nor any quantity derived therefrom shows a skewed velocity profile.

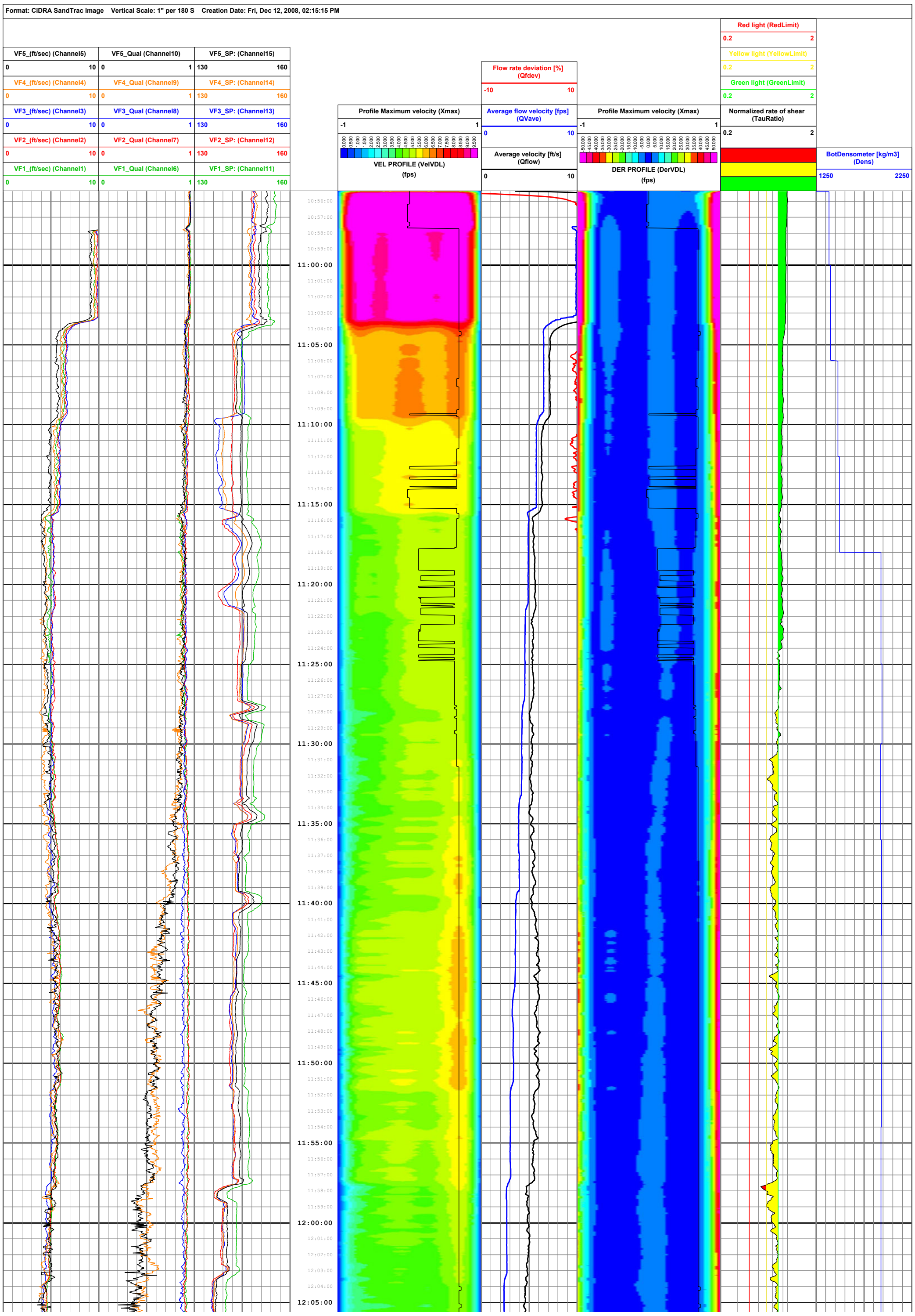

Figure 4: Mono-dispersed sand

4 FIELD CASE: HYDRO-TRANSPORT

In addition to laboratory flow loop data, field data will be presented in the same format. The cases to be shown cover one and the same location of a hydro-transport line flowing water, sand and bitumen; but at different times. Even though visually not much appears to be different it will be shown that the ζ ratio picks up a marked difference between the two.

Field data from one and the same hydro-transport line flowing a mixture of water, rocks, coarse and fine sand is shown in Figure 5 and 6. Observe how at the beginning the ζ ratio is nearly equal to 1 indicative of a symmetrical velocity profile.

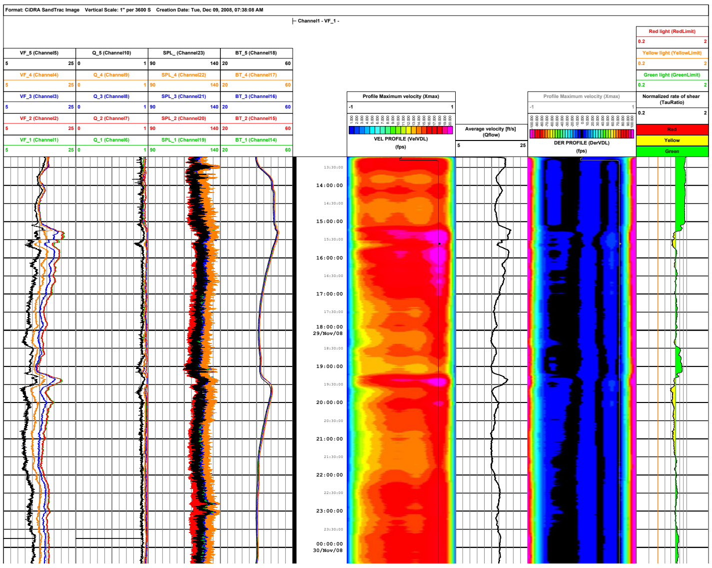

Figure 5: Hydrotransport line results, safe conditions

Visually there is little difference between the two sets of data. Still the ζ ratio picks up a marked difference between the two time periods.

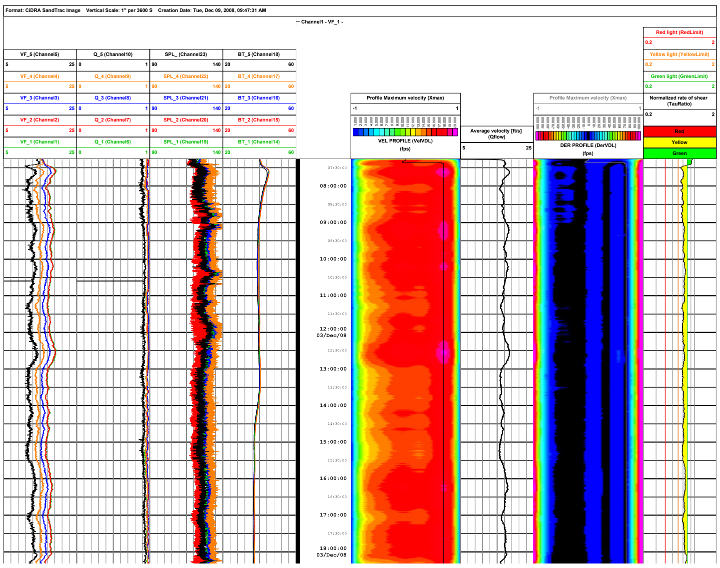

Figure 6: Hydrotransport line results, warning conditions

5 CONCLUSIONS AND RECOMMENDATIONS

It is concluded that a *5 sector array* velocity profiler can measure the velocity profile of heaveily skewed, stratified flows in polydisperse slurries. In monodisperse slurries, where the particulate material is classified and the finer fraction is used, the measured velocities do not show any skew or stratification.
In hydro-transport the sanding metric that is proposed, i.e. the ζ ratio shows a clear differentiation between two different cases of a meter being installed on the same line at the same location. Visually the raw data looks similar but a subtle difference does exist.

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