ABSTRACT 21

On-line measurement of rheological parameters for feedback control of thickeners

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ABSTRACT

Thickened tails and pastes often exhibit a yield point and a plastic viscosity which varies with the shear rate. The flow of such fluids through pipelines differs markedly from the flow of more dilute slurries in at least two aspects. First, the variation of the shear stress at the wall with the shear rate at the wall will change with varying rheological parameters of the fluid. Second, the mechanism by which mechanical energy propagates between length scales to final dissipation will change with the degree of turbulence exhibited by the flow.

Whereas shear stress at the wall is trivially related to pressure gradient and, thus, easy to measure, the shear rate at the wall is much harder to evaluate as this would require a velocity profile near the wall. Measurements of velocity profiles in turbulent flow of waterborne slurries and in flows of Bingham fluids show little variation despite their differences. In homogenous fluids, however, the ratio of fractional change in wall shear stress to the fractional change in dimensionless rate of flow is firmly related to the shear rate at the wall. Robust flow measurements combined with reliable differential pressure data, therefore, allow on-line determination of fluid rheological parameters necessary for feedback control.

In homogenous, isotropic turbulence the energy spectrum of eddies in high Reynolds number flow is given by the Kolmogorov 5/3 law. Deviation from this law will indicate a change in the degree of turbulence possibly as a result of a change in the rheological parameters of the fluid. Array type measurement of the rate of flow, e.g. by sonar methods, enables the evaluation of the energy spectrum of eddies. Thus, the energy spectrum of the rate of dissipation of mechanical energy is used directly to infer information about the rheological parameters of the flow.

INTRODUCTION

Thickened Tails (TT) is a rheological fluid characterized by a non-Newtonian constitutive equation. Often a Bingham model or a Herschel-Bulkley model is applied to model the rheological behaviour of TT. Bingham fluids exhibit a yield point. Wall shear stresses below the yield point will not result in flow. Otherwise, the shear stress increases linearly with the shear rate. Herschel-Bulkley fluids also exhibit a yield point, but the shear stresses increase either super-linearly or supra-linearly with the shear rate. Herschel-Bulkley fluids are therefore capable of modelling dilatant or pseudo-plastic behaviour. A comparison of the various constitutive equations for TT is given in Figure 1. This figure is to scale and representative values for TT were used. For reference the black line represents water at room temperature, which is purely a Newtonian fluid

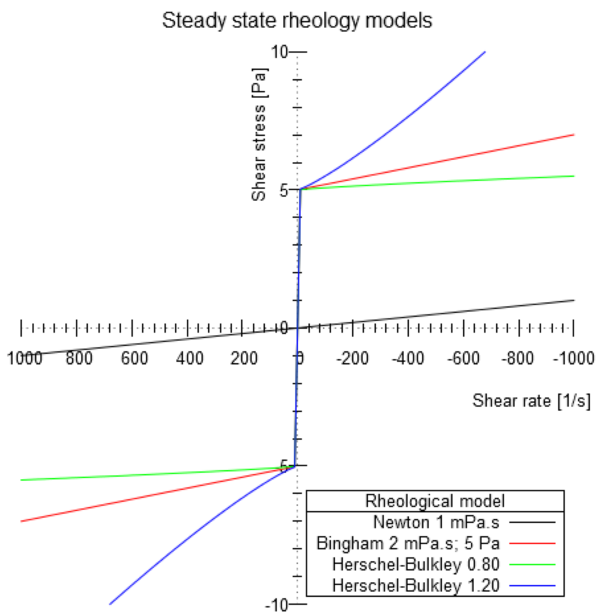

Figure 1 Comparison of various rheological models

In order to control thickeners producing TT, two conceptually different methods may be used. In feed-forward control a variety of parameters of the feed of the thickener are measured and the operation of the thickener is adjusted to accommodate variations in the feed. Parameters of the feed of interest for feed-forward control are the flow rate, the density and the particle size distribution of the feed flow. In feed-back control a number of quantities of the product of the thickener are measured and the operation of the thickener is adjusted to maintain such parameters of interest constant or within certain pre-set bounds. The quantities of interest for feed-back control are rheological parameters of TT, such as the yield point and the plastic viscosity, which can be derived from shear stress and shear rate. There is currently no known technology for on-line, real-time monitoring of such rheological parameters. In practice, samples are taken and measured using a viscometer in a laboratory.

According to Figure 1, assessing TT rheological parameters should be possible by simultaneous measurement of wall shear stress and wall shear rate. Monitoring the change in rheology with varying thickener operating parameters would be possible if both the wall shear stress and the wall shear rate in the tailings line were available. It is very easy to infer the wall shear stress from the pressure gradient, but it is much harder to find the wall shear rate.

Alternatively, the transition from a non-fully sheared Bingham like plug flow (which is non-turbulent) to fully developed turbulent flow may be monitored by measuring the decay of turbulent energy coupled with vorticity. This latter method effectively uses the fact that energy dissipation in turbulent flow occurs at a rate much higher than in the non-turbulent flow of TT.

In summary three conceptually different monitoring methods may be evaluated:

1. Monitor the TT by vertical velocity profiling.
2. Measure the wall shear rate and cross plot this versus wall shear stress.
    a. By measuring the (vertical) velocity profile.
    b. By assuming a specific rheological model.
    c. By inspection of the slope of a cross plot of flow and pressure drop.
3. Derive the degree of turbulence from the energy spectrum of vorticity.

A number of the above three methods will be investigated in the next chapter. Methods 1 and 2a require a velocity profile to be measured. Method 3 requires an array method in order to evaluate the spectral energy decay. Method 2b assumes a specific rheological model and is thus prone to being in error. Method 2c cannot provide real time data.

METHODOLOGY

The wall shear stress plays an important role in pipe flow of rheological fluids. An integral momentum balance gives for the wall shear stress τ_R:

$$\tau_R = \frac{R}{2}\frac{dp}{dx} \quad (1).$$

Where R is the pipe's inside radius and dp/dx is the pressure gradient along the axial dimension of the pipe. The shear stress at the wall is also the maximum stress in the fluid. The profile of the shear stress in pipe flow is always linear as the momentum balance equation is a differential equation of first order.

The Buckingham—Reiner equation

A Bingham fluid in a pipe will start to flow once the wall shear stress exceeds the yield point τ_0 of the fluid. In such cases the relation between the shear rate and the shear stress is given by:

$$\tau_{yx} = -\mu_0 \frac{\partial v_x}{\partial y} \pm \tau_0 \quad (2).$$

Here μ_0 is the plastic viscosity of the fluid. If the wall shear stress is less than the yield point, the shear rate $\partial v_x/\partial y$ is zero everywhere. For flow of this type it is possible (Bird et.al., 1960) to express the volumetric rate of flow Q to the other quantities:

$$Q = \frac{\pi R^3}{4\mu_0}\tau_R\left(1 - \frac{4}{3}\frac{\tau_0}{\tau_R} + \frac{1}{3}\left(\frac{\tau_0}{\tau_R}\right)^4\right) \quad (3).$$

Introducing the non dimensional rate of flow y:

$$y = \frac{4Q}{\pi R^3} \frac{\mu_0}{\tau_0} \qquad (4),$$

and the ratio of the yield point to the wall shear stress as:

$$x = \frac{\tau_0}{\tau_R} \qquad (5)$$

The equation (3); for the volumetric rate of flow may be cast in the form of a quartic:

$$x^4 - (4 + 3y)x + 3 = 0 \qquad (6)$$

The solution of this equation is given in Figure 2. The point with x =1 and y = 0 is the point identifying no flow as the wall shear stress is equal to the yield point.

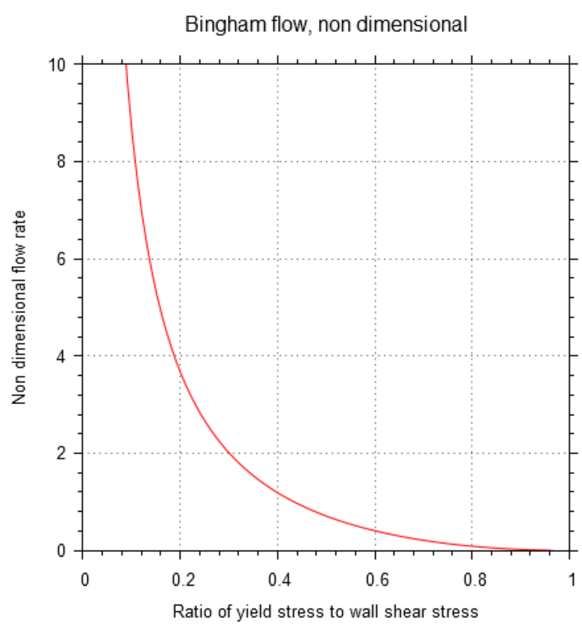

Figure 2 Solution of the Buckingham—Reiner equation

The simple theory above suffices to compute material properties of the Mature Fine Tailings (MFT) from measured rate of flow and pressure gradient. The equation for the volumetric rate of flow, equation (3), relates rate of flow, pressure gradient, yield point and plastic viscosity. If we were to measure, simultaneously, the rate of flow and the pressure gradient, then either the yield point or the plastic viscosity of the fluid could be calculated with the help of equation (3). Thus, if the yield point of the fluid were known and the pressure gradient is measured, the non-dimensional quantity x can be substituted into equation (6) and y can be solved for.

With a known, measured value of the rate of flow, the plastic viscosity can thus be retrieved, and the wall shear rate γ_R then follows as:

$$\gamma_R = \frac{\tau_R - \tau_o}{\mu_o} \quad (7).$$

The drawback of this method to find the wall shear rate is that it assumes the existence of Bingham flow, and it requires that at least one parameter of the two parameter Bingham model is known a-priori. In more complicated three parameter rheological models this becomes worse.

Yet, the theory presented lends itself well to designing for an experiment where pressure gauges and velocity (profile) meters must be placed and specified.

The Rabinowitsch—Weissenberg equation

There is a way to derive the wall shear rate from measurement of the rate of flow and the wall shear stress only. The Rabinowitsch equation was first derived by Weissenberg in 1929 (Bird et.al., 1960) and is an expression of the wall shear in terms of the slope of the rate of flow versus wall shear stress on a double log plot. The derivation of this equation starts with the general expression for the volumetric flow rate Q from some velocity profile v(r):

$$Q = 2\pi \int_0^R v(r) r dr \quad (8),$$

Integration by parts then results in:

$$Q = -\pi \int_0^R r^2 \left(\frac{dv}{dr} \right) dr \quad (9).$$

Upon a coordinate transformation to remove the radial coordinate in favour of the shear stresses:

$$\frac{r}{R} = \frac{\tau}{\tau_R} \quad (10),$$

it follows that:

$$\left(-\frac{dv}{dr} \right) = \gamma_R = \frac{1}{\pi R^3 \tau_R^2} \frac{d}{d\tau_R} \left(\tau_R^3 Q \right) \quad (11).$$

Simple manipulations then lead to the result:

$$\gamma_R = \frac{Q}{\pi R^3} \left(3 + \frac{d \log Q}{d \log \tau_R} \right) \quad (12).$$

This equation is the Rabinowitsch-Weissenberg equation. Its derivation requires no assumptions as to the constitutive equation of the fluid. The equation holds true for homogenous, isothermal flow without momentum mixing.

Since the computation of the wall shear rate by equation (12) requires that the flow rate and pressure gradient are cross plotted on logscales, a single measurement of flow rate and pressure gradient cannot give the wall shear rate. Therefore, application of the Rabinowitsch equation can only be successful on historical data looking back a certain amount in time. The time lag thus introduced is not ideal for control applications.

The Kolmogorov theory of homogenous isotropic turbulence.

In the Kolmogorov theory (Davidson, 2009) the wavenumber k is used in preference over a frequency because the wavenumber is more closely related to the breakdown of vortical structures in the flow. Such vortical structures have a maximum length scale equal to the pipe diameter and the maximum wavelength that can occur is therefore equal to the pipe diameter.

The Kolmogorov theory postulates that the energy of vortical eddies is distributed over ever smaller length scales without any dissipation until the smallest length scale (appropriately called the Kolmogorov length scale) is reached where the energy is finally dissipated into heat. The Kolmogorov length scale, time scale, velocity scale and the derived shear stress scale are given below in Table 1 for a typical MFT.

Table 1 Kolmogorov scales

	Formula	**Value**	**Unit**
Length	$\sqrt[4]{\nu^3/\epsilon}$	0.2	mm
Time	$\sqrt{\nu/\epsilon}$	5	ms
Velocity	$\sqrt[4]{\nu\epsilon}$	4	cm/s
Shear stress	$\sqrt{\varepsilon/\nu}$	200	1/s

Where υ is the kinematic viscosity of the fluid and ε is the rate of dissipation of energy per unit mass. It is concluded that the Kolmogorov length scale is much larger than the typical particle sizes of the fines (< 44 μm) that make up MFT. It is only when such fines are flocculated that this length scale may be approached. It is also concluded that the Kolmogorov length scale is much smaller than the length scale to which a sensor array with a spacing of 61 mm is sensitive to. Apparently, measured vortical power spectra by a sonar array are thus not affected by either the small scale eddies nor by the large scale effects, which are on the order of the pipe diameter.

The theory of homogenous isotropic turbulence as developed by Kolmogorov predicts that in this range the spectrum of vortical eddies should be a unique function of the wavenumber scaled by the pipe diameter. In the inertial subrange, where the length scale is much larger than the Kolmogorov length scale and much smaller than the pipe diameter the spectrum follows a power law:

$$P(k) = c\varepsilon^{2/3}k^{-5/3} \quad (13).$$

Here c is a constant, ε is the rate of energy dissipation per unit mass, which can be cast into the product of velocity V times pressure gradient dp/dx:

$$\varepsilon = \frac{1}{\rho}V\frac{dp}{dx} \quad (14).$$

The sensor array that makes up a sonar flow meter is well suited to infer the vortical power spectra directly. It is thus possible to evaluate the energy dissipation rate in the fluid directly without the need for differential pressure. Furthermore, since the power spectrum P(k) is measured, this allows an estimate of the energy dissipation at various length scales of interest.

Measured vortical power spectra

The sonar array measures the vortical power distribution along the line of the volumetrically averaged velocity in the wavenumber-frequency diagram. Plotting this power distribution versus the wavenumber for a number of pipe sizes, we see a striking pattern as given in Figure 3, left hand panel.

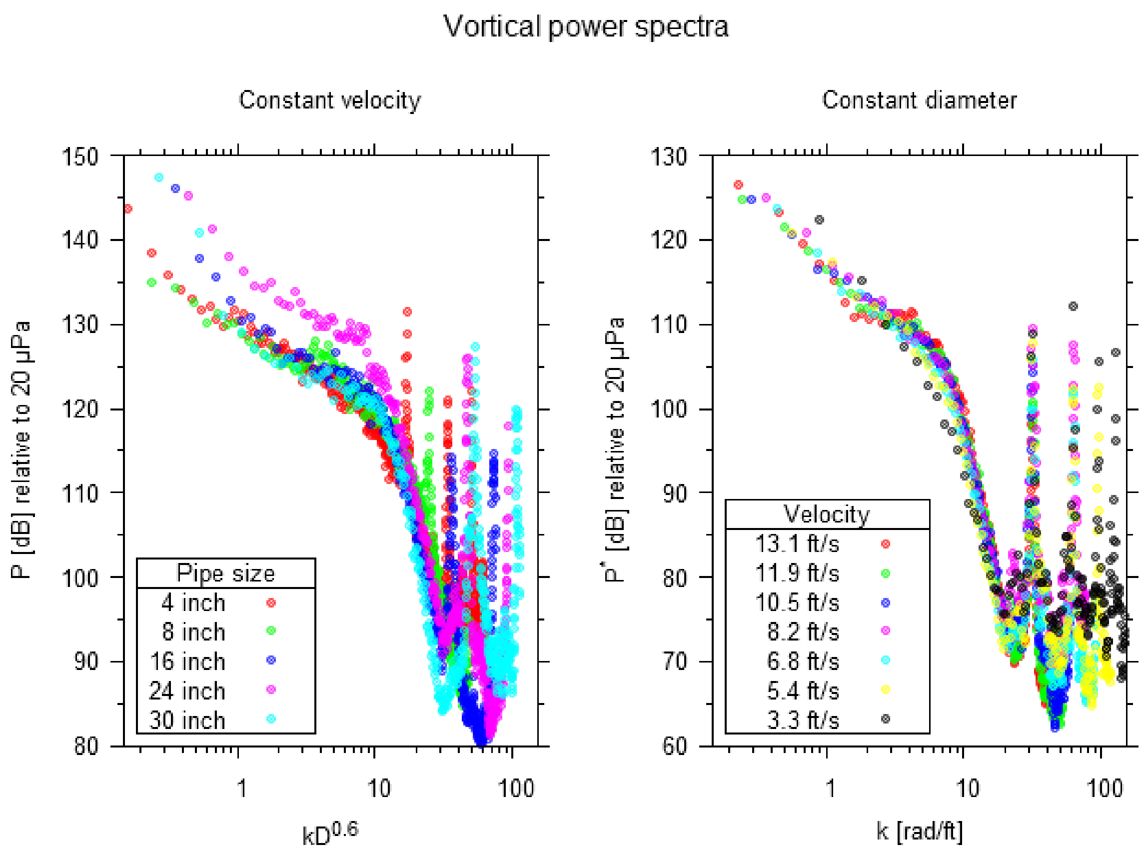

Figure 3 Vortical power spectra

The various peaks in this plot are due to spatial aliasing, which results because of the finite and small spatial sampling. Most of the data collapses to one curve as is predicted by the Kolmogorov theory.

Vortical spectra for constant pipe diameter but different flow velocity are shown in Figure 3 on the right hand panel. The range of velocities covers about one decade starting at what is close to the minimum measureable velocity in turbulent flow. Much like the previous spectra for constant velocity, the spectra follow the Kolmogorov prediction over a wide range of wavenumbers.

Experimental field data on thickened tails

We present field data taken on a pipe reactor of length L1 where MFT is treated by polymer flocculant in order to release water and allow the MFT to dry. The treatment of MFT is an important topic for oil sands water and land reclamation. In a sense the treatment of MFT can be viewed as the opposite of the action of a thickener producing TT. The advantage of MFT is that the treatment occurs within a reasonably short pipeline. As a result at the start point, the MFT is still in its native state, and at the end point, just upstream of the T-off point to the drying cells, the MFT particulate matter is settling and the rheology (and as a result the flow regime) has changed. The pipe reactor was instrumented with triple sonar based velocity profile meters. The distance between the meters was such that given the length of the pipe reactor and the average flow velocity the chances of observing the changes in the rheology were as large as possible.

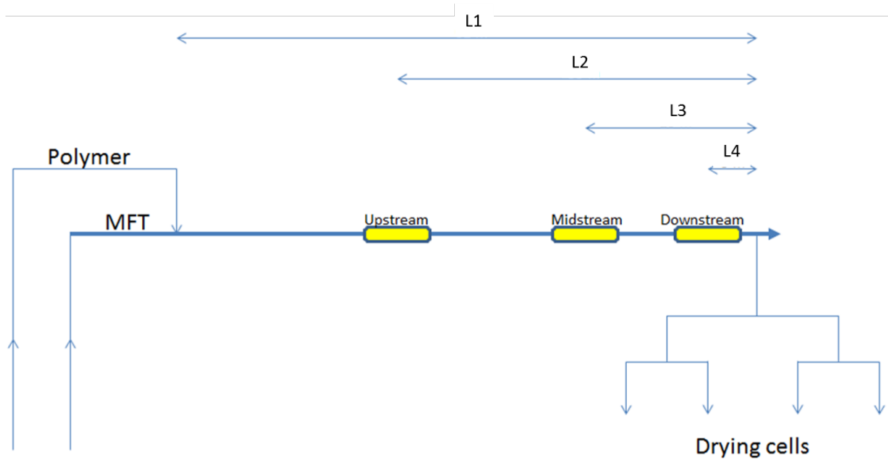

Figure 4 Experimental field setup.

Unfortunately, in this field test differential pressure in the pipe reactor was not measured. Consequently, the wall shear stress variation cannot be obtained. During the course of the experiment, it became obvious that the differences in velocity profile between the flow regime of untreated MFT and fully treated MFT are too small to be meaningful.

The sonar based velocity meters (O'Keefe, 2009), however, measure the vortical power associated with the coherent power of vortices convecting through the meter's sensor array. Vortical power spectra are calculated in the meter's array processing and a figure of merit, called vortical power quality, is derived. High vortical power quality indicates stable, low loss propagation of coherent power. Low vortical power quality indicates higher loss of coherency.

In Figure 5 we plot the temporal variation of vortical power quality for all three (upstream, midstream and downstream) velocity profile meters for each of the 5 oriented sensor arrays in each meter. The top panel displays the vortical power quality for the top-most sensor array. The bottom panel displays the vortical power quality for the bottom-most sensor array. The panels in between relate to the other three sensor arrays located at the horizontal, 45° and 135° measured from the top, positions. In each panel the red, green and blue trace belong to the upstream, midstream and downstream meter respectively.

Evidently, around 16:00 there is a marked change occurring. Whereas in the measured velocity data this change is hardly noticeable, the vortical power quality shows a marked difference in both the range of the quality as well as in the variation (fluctuation) thereof. At 16:00 the injection of polymer was stopped, and the flow of MFT then reverts back to laminar for the entire pipe reactor.

The precise difference in the range and variation of the vortical power quality becomes clearer if we make quality distributions by binning. For the period of 12:00 to 14:00 (turbulent flow) and the period 18:00 to 20:00, this is presented in Figure 6. Each column of panel plots in this figure represents a meter from upstream on the left, midstream in the middle and downstream on the right. Each row of panel plots represents a sensor array, from top array at the top to bottom array at the bottom. The darker colours refer to the 12:00 to 14:00 time period, i.e. turbulent flow. The lighter colours refer to the 18:00 to 20:00 time period, i.e. laminar flow.

Each panel plot also shows the cumulative distribution as a solid black line. No attempt was made to differentiate these between the laminar and turbulent flow regime as it is obvious which one belongs to which. Likewise a dotted black line represents a kernel density estimate of the histogram values.

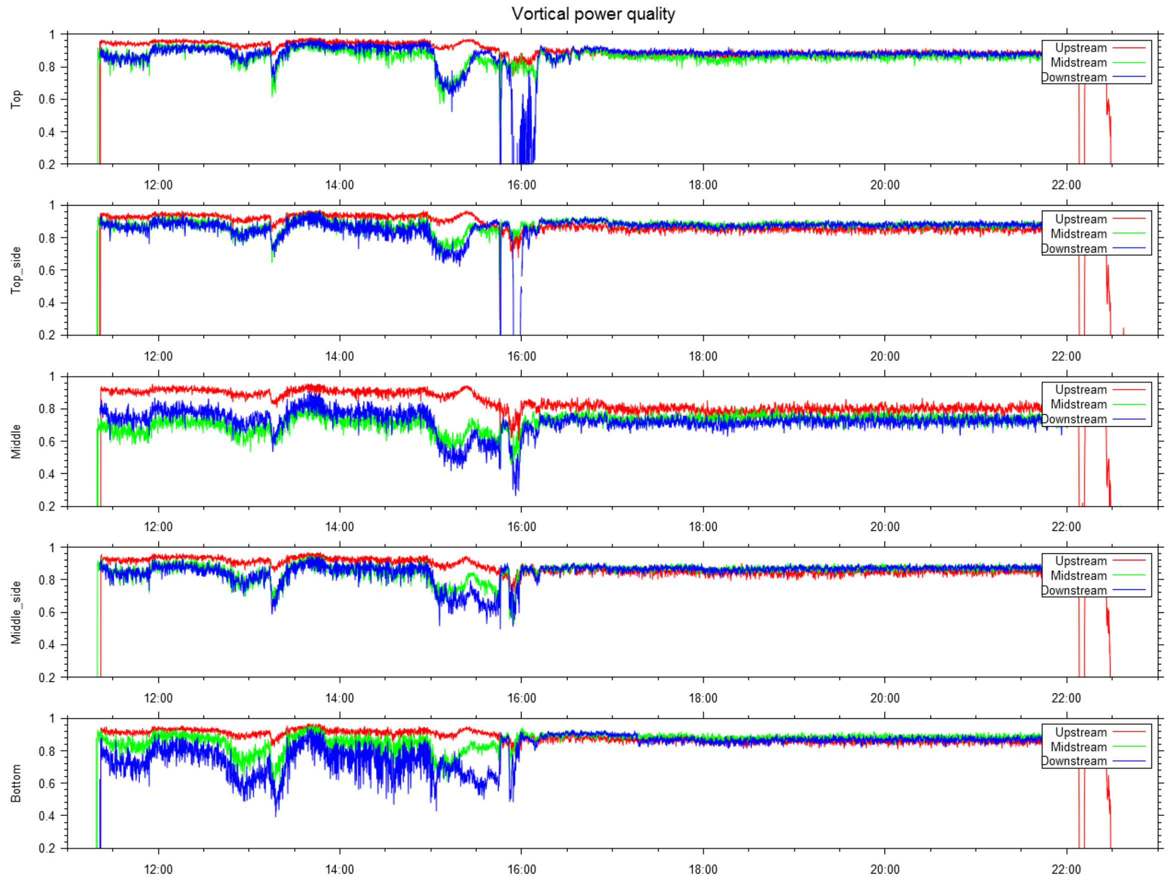

Figure 5 Vortical power quality data in turbulent and laminar flow of thick tailings.

Evidently, the vortical power quality in turbulent flow shows a trend towards lower values and larger spreads as the flow progresses from upstream to downstream and as the location in the pipe varies from top to bottom. Contrariwise, in laminar flow the vortical power quality remains at a fixed location and does not spread. The middle sensor array shows consistently lower mean value in both turbulent and laminar flow. It is presently not known why this is the case.

Thus, the statistical properties of vortical power quality can be used to discriminate between not only the flow regime (laminar or turbulent) the same properties can be used to discriminate between the various instances of turbulent flow as the MFT continues to react with polymer, which alters its rheological properties. For instance the ratio of the second and first moment can be used to infer the degree of turbulence of the flow, which is useful property to control the polymer injection.

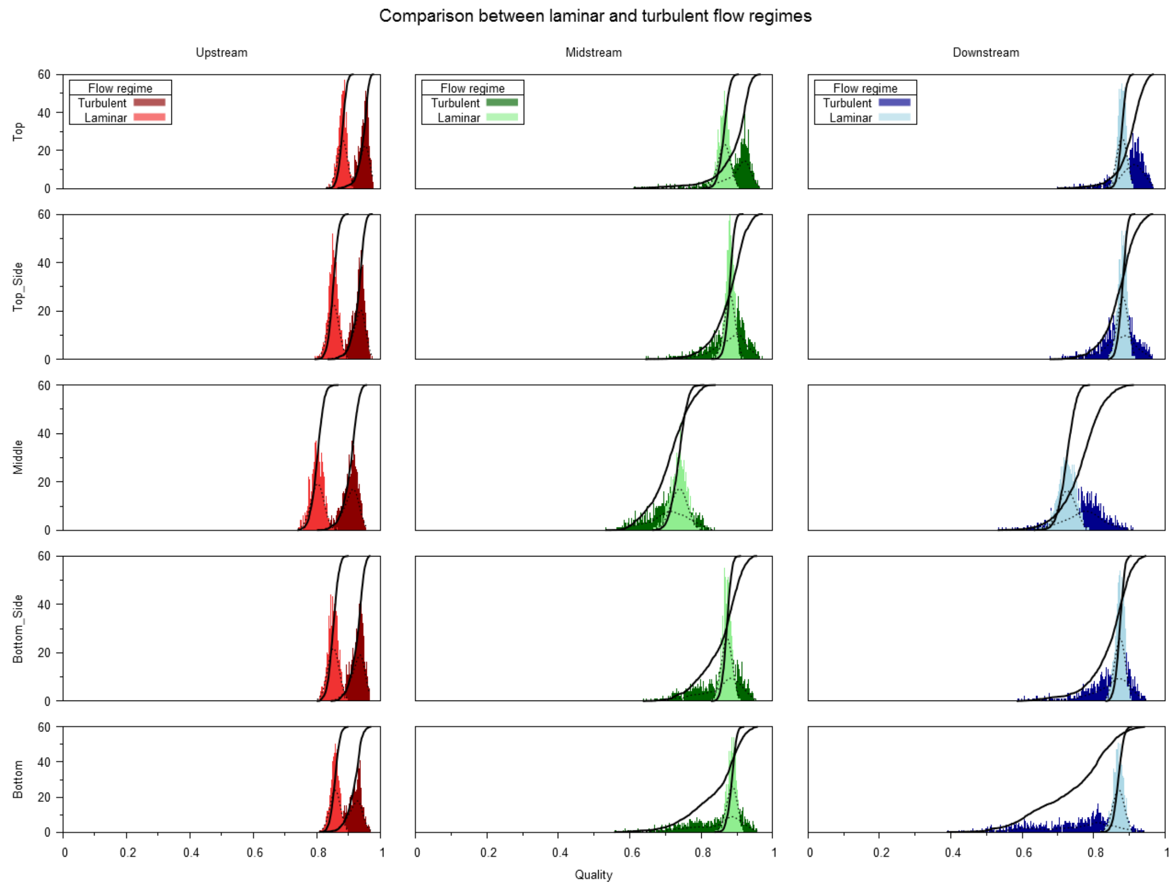

Figure 6 Vortical power quality distributions for turbulent and laminar thick tails.

RESULTS AND DISCUSSION

Because of the unavailability of differential pressure in the pipe reactor, any method for on-line rheology that requires wall shear stress could not be applied. During the course of the work, it became clear that such differences as there must be between a fully developed turbulent flow profile and a flow profile of a Bingham type of fluid in laminar flow are too small to be observed. In parallel, however, the distribution of vortical power quality shows marked changes between different flow regimes. The statistical properties of vortical power quality can be used to infer a parameter that allows control of the reagent rate.

CONCLUSION

Using sonar based meters, it is possible to distinguish between the flow regime of untreated MFT flowing in laminar flow as a Bingham fluid and the flow regime of treated MFT that is transitioning to turbulent flow. The degree of turbulence (in a qualitative sense) may be inferred from the statistical moments of the distribution of the vortical power quality, which is measured by the sonar meter.

ACKNOWLEDGEMENTS

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NOMENCLATURE

Q	Volumetric rate of flow
p	Pressure
R	Radius of pipe or wall coordinate
x	Axial coordinate, dimensionless wall shear stress
y	Dimensionless rate of flow
v	Velocity of flow
τ	Shear stress or yield point
μ	Plastic (dynamic) viscosity
γ	Shear rate
r	Radial coordinate
c	Constant
k	Wavenumber
ε	Rate of energy dissipation per unit mass
l	Kolmogorov length scale
υ	Kinematic viscosity

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