Accurate Liquid Phase Density Measurement of Aerated Liquids using Speed of Sound Augmented Coriolis Meters

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Abstract
A methodology is described to improve the accuracy of vibrating-tube-based density measurements of aerated liquids. In many applications, density measurements are employed to determine compositional information of process liquids. For most density measuring devices, the presence of a small, but unknown, quantity of entrained gaseous phase within the process mixture can introduce significant errors in both the measured mixture density as well as the interpreted density of the liquid phase.

This paper describes an approach to measuring fluid density which couples a sonar-based speed-of-sound measurement with vibrating-tube-based density measurement, commonly used in coriolis mass and density meters, to determine the density of aerated liquids. It is well known that the accuracy of coriolis meters can be significantly degraded with the aeration of the process fluid. Augmenting the output of the coriolis meter with a speed of sound measurement provides a novel approach to improved density measurement for aerated fluids in two ways. Firstly, sound speed based gas volume fraction measurement provides a first-principles-based, real time measurement of the gas volume fraction and compressibility of the aerated process fluid. Secondly, the sound speed of the process fluid is used to compensate for the effect of the increased compressibility and inhomogeniety of aerated mixtures on the output of the coriolis density measurement.

To illustrate the fundamental ways in which aeration impacts vibrating-tube density measurements, a simplified, lumped parameter model for the effects of aeration in vibrating tubes is developed. The model illustrates that the effects of aeration can be attributed to at least two independent mechanisms; 1) the density inhomogeniety of discrete gas bubbles and 2) increased mixture compressibility due to aeration. Analytical results are supported by experimental data which suggest that augmenting the density measurements from the coriolis meter with a sound speed measurement significantly enhances the ability determine the density of aerated liquids with an accuracy that approaches that for non-aerated mixtures.

Introduction

Coriolis mass flow and density meters are considered the flow metering solution of choice for many precision flow applications. Since their introduction to the mainstream flow metering community in the early 1980's, coriolis meters have grown into one of largest and fastest growing market segments, representing roughly $400 million annual sales on approximately 100,000 units. The success of coriolis meters has been attributed to many factors, including its accuracy, reliability, and ability to measure multiple process parameters, including mass flow and process fluid density. However, despite this long list of attributes, coriolis meters have significant limitations regarding aerated liquids. Although the mass flow rate and process fluid density measurements determined by coriolis meters are derived from independent physical principles, the accuracy of both are significantly degraded with the introduction of small amounts of entrained gases. Although the mechanisms responsible for the degradation of the measurements in the presence of aeration are similar, the scope of this paper is limited to assessing and enhancing the accuracy of coriolis meter-based density measurements.

Coriolis Density Measurement

Although the specific design parameters of coriolis meters are many and varied, all coriolis meters are essentially aeroelastic devices. Aeroelasticity is a term developed in the aeronautical sciences that describes the study of dynamic interaction of coupled fluid dynamic and structural dynamic systems, for example the static and dynamic response of an aircraft under aerodynamic forces. Coriolis flow meters rely on characterizing the aeroelastic response of fluid-filled, vibrating flow tubes to determine both the mass flow rate and process fluid density measurements.

The physical principle used to determine process fluid density in a Coriolis meter is similar to that used in vibrating tube density meters. In these devices, the density of the process fluid is determined by relating the natural frequency of a fluid-filled tube to the density of the process fluid. To illustrate this principle, consider the vibratory response of a vacuum-filled flow tube.

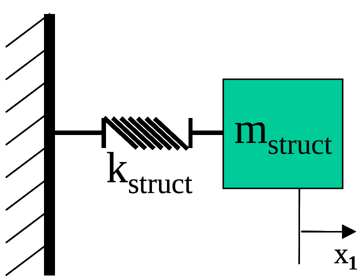

Figure 1: Lumped Parameter Model for Tube Oscillation

In this model, shown schematically in Figure 1, the frequency of oscillation is given by the ratio between the effective stiffness (K_{struct}) of the tubes and the effective mass (m_{struct}) of the tubes.

$$f_{nat} = \frac{1}{2\pi}\sqrt{\frac{K_{struct}}{m_{struct}}}$$

Introducing fluid to the tube changes the natural frequency of the oscillation. Under a quasi-steady and homogeneous model of the fluid, the primary effect of the fluid is to mass-load the tubes. The fluid typically has a negligible effect on the stiffness of the system. Thus, within the framework of this model, the mass of the fluid is added directly to the mass of the structure, as shown schematically in Figure 2.

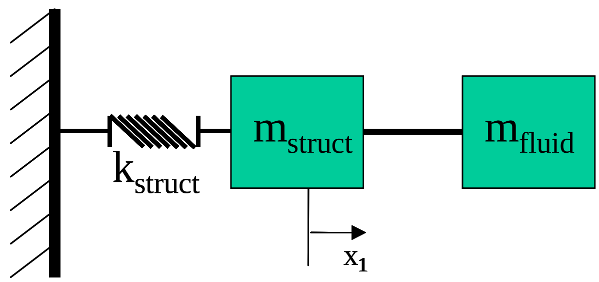

Figure 2: Lumped Parameter Quasi-steady, Homogeneous Model for Fluid-Filled Tube Oscillation

The mass of the fluid in the tube is proportional to fluid density, and therefore, the naturally frequency decreases with increasing fluid density as described below:

$$f_{nat} = \frac{1}{2\pi}\sqrt{\frac{K_{struct}}{m_{struct} + \beta\rho_{fluid}}}$$

where β is a calibrated constant related to the geometry and vibratory characteristic of the vibrating tube.

Rearranging, the algebraic relation between the measured natural frequency of the vibrating tube and the density of the fluid within the tube can be written as follows.

$$\rho_{fluid} = \frac{1}{\beta}\left(\frac{K_{struct}}{(2\pi)^2 f_{nat}^2} - m_{struct}\right)$$

Defining the ratio between the effective mass of the fluid to that of the structure as α, the natural frequency of the fluid loaded tubes is given by:

$$f_{nat} = f_s \sqrt{\frac{1}{1+\alpha}} \quad \text{where } \alpha \equiv \frac{m_{fluid}}{m_{struct}}$$

This basic framework provides an accurate means to determine process fluid density under most operating conditions. However, some of the fundamental assumptions regarding the interaction of the fluid and the structure can deteriorate under different operating conditions. Specifically, this paper addresses how the aerated fluids in oscillating tubes behave differently from single phase fluids in two important ways; increased compressibility, and fluid inhomogeneity.

Fluid Compressibility

It is well known that most aerated liquids are significantly more compressible than non-aerated liquids. Compressibility of a fluid is directly related to the speed of sound and density of the fluid.

Mixture density and sound speed can be related to component densities and sound speed through the following mixing rules which are applicable to single phase and well-dispersed mixtures and form the basis for speed-of-sound-based entrained air measurement (Gysling, 2003).

$$\kappa_{mix} = \frac{1}{\rho_{mix} a^2_{mix_\infty}} = \sum_{i=1}^{N} \frac{\phi_i}{\rho_i a_i^2}$$

where $\rho_{mix} = \sum_{i=1}^{N} \rho_i \phi_i$ and κ_{mix} is the mixture compressibility, and ϕ_i is the component volumetric phase fraction.

Consistent with the above relations, introducing air into water dramatically increased the compressibility of the mixture. For instance, at ambient pressure, air is approximately 25,000 times more compressible than water. Thus, adding 1% entrained air increases the compressibility of the mixture by a factor of 250. Conceptually, this increase in compressibility introduces dynamic effects that cause the dynamic of behavior of the aerated mixture within the oscillating tube to differ from that of the essentially incompressible single-phase fluid.

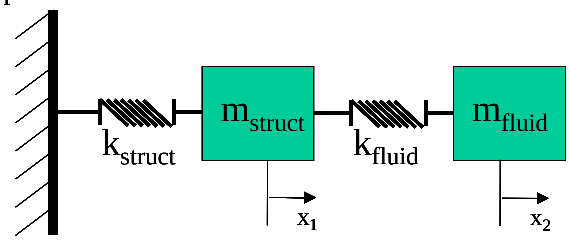

Figure 3: Lump Parameter Model for the Oscillation of a Tube Filled with a Compressible Fluid

The effect of compressibility of the fluid can be incorporated into a lumped parameter model of a vibrating tube as shown schematically in Figure 3. The stiffness of the spring represents the compressibility of the fluid. As the compressibility approaches zero, the spring stiffness approaches infinity and the model becomes equivalent to that presented in Figure 2.

As before the effective mass of the fluid is proportional to the density of the fluid and the geometry of the flow tube. The natural frequency of the first transverse acoustic mode in a circular duct can be used to estimate an appropriate spring constant for the model

$$f = \frac{1.84}{\pi D} a_{mix} = \frac{1}{2\pi}\sqrt{\frac{K_{fluid}}{m_{fluid}}}$$

Note that this frequency corresponds to a wavelength of an acoustic oscillation of approximately two diameters, i.e., this transverse mode is closely related to a “half wavelength” acoustic resonance of the tube. Figure 4 shows the resonant frequency of the first transverse acoustic mode of a one-inch tube as a function of gas volume fraction for air entrained in water at standard temperature and pressure. For low levels of entrained air the frequency of the first transverse acoustic mode is quite high compared to the typical structural resonant frequencies of coriolis meters of 100 Hz, however, the resonant acoustic frequency decreases rapidly with increased levels of entrained air.

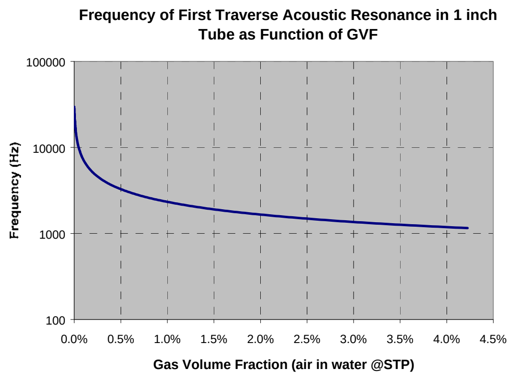

Figure 4: Natural Frequency of First Acoustic Cross Mode as Function of Gas Volume Fraction for 1 inch Tube for Air in Water @STP

In characterizing aeroelastic systems, it is often convenient to define a reduced frequency parameter to gauge the significance of the interaction between coupled dynamic systems. For a vibrating tube filled with fluid, a reduced frequency can be defined as a ratio of the natural frequency of the structural system to that of the fluid dynamic system.

$$f_{red} = \frac{f_{struct} D}{a_{mix}}$$

Where f_{struct} is the natural frequency of the tubes in vacuum, D is the diameter of the tubes, and a_{mix} is the sound speed of the process fluid. For this application, as the reduced frequency becomes negligible compared to 1, the system approaches quasi-steady operation. In these cases, models, which neglect the compressibility of the fluid, such as that shown in Figure 3, are likely to be suitable. However, the effects of unsteadiness increase with increasing reduced frequency. For a given coriolis meter, mixture sound speed has the dominant influence of changes in reduced frequency. Figure 5 shows the reduced frequency plotted as a function of entrained air for a one-inch diameter tube with a structural natural frequency of 100 Hz. As shown, the reduced frequency is quite small for the non-aerated water; however, builds rapidly with increasing gas volume fraction, indicating that the significance of compressibility increases with gas volume fraction. However, when considering coriolis meters of varying design parameters, increases in tube natural frequency or tube diameter will increase the effects of unsteadiness for a given level of aeration.

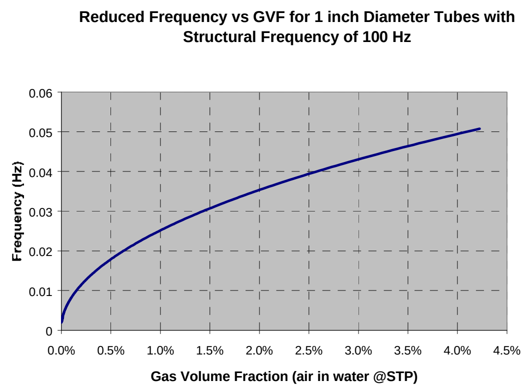

Figure 5: Reduced Frequency of First Acoustic Cross Mode as Function of Gas Volume Fraction for 1 inch Tube for Air in Water @STP

Fluid Inhomogeneity

In additional to dramatically increasing the compressibility of the fluid, aeration introduces inhomogeneity to the fluid. For flow regimes in which the gas is entrained in a liquid-continuous flow field, the first–order effects of the aeration can be modeled using bubble theory. By considering the motion of an incompressible sphere of density of ρ_0 contained in an inviscid, incompressible fluid with a density of ρ and set into to motion by the fluid, Landau and Lipshitz, show that the velocity of the sphere is given by:

$$V_{sphere} = \frac{3\rho}{\rho + 2\rho_0} V_{fluid}$$

For most entrained gases in liquids, the density of the sphere is orders of magnitude below that of the liquid and the velocity of bubble approaches three times that of the fluid.

Considering this result in the context of the motion of a sphere in a cross section of a vibrating tube, the increased motion of the sphere compared to the remaining fluid must result in a portion of the remaining fluid having a reduced level of participation in oscillation, resulting in a reduced, apparent system inertia.

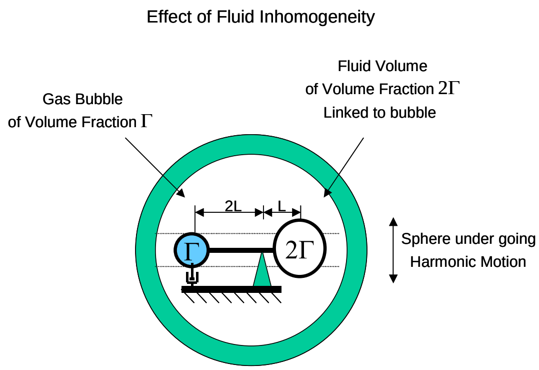

Figure 6: Lump Parameter Model for the Inertia of an Incompressible Aerated Fluid contained in Tube undergoing Harmonic Motion

Figure 6 illustrates a lumped parameter model for the effects of inhomogeniety in the oscillation of an aerated-liquid-filled tube. In this model, a gas bubble of volume fraction Γ is connected across a fulcrum to a compensating mass of fluid with volume 2 Γ. The fulcrum is rigidly connected to the outer tube. The effects of viscosity can be modeled using a damper connected to restrict the motion of the gas bubble with respect to the rest of the liquid and the tube itself. The remaining volume of liquid in the tube cross section (1-3 Γ) is filled with an inviscid fluid. In the inviscid limit, the compensating mass of fluid (2Γ) does not participate in the oscillations, and the velocity of the mass-less gas bubble becomes three times the velocity of the tube. The effect of this relative motion is to reduce the effective inertia of the fluid inside the tube to (1-3 Γ) times that presented by a homogeneous fluid-filled the tube. In the limit of high viscosity, the increased damping constant minimizes the relative motion between the gas bubble and the liquid, and the effective inertia of the aerated fluid approaches 1-Γ. The effective inertia predicted by this model of an aerated, but incompressible, fluid oscillating within a tube agrees with those presented by (Hemp, et al, 2003) in the limits of high and low viscosities.

Combined Lumped Parameter Model

Models were presented with the effects of aeration on vibrating tube density meters in which the effects of compressibility and inhomogeniety were addressed independently. Figure 7 shows a schematic of a lumped parameter model that incorporates the effects of compressibility and inhomogeniety using the mechanism-specific models developed above. The purpose of this model is to illustrate trends and parametric dependencies associated with aeration, it is not intended as a quantitative predictive tool.

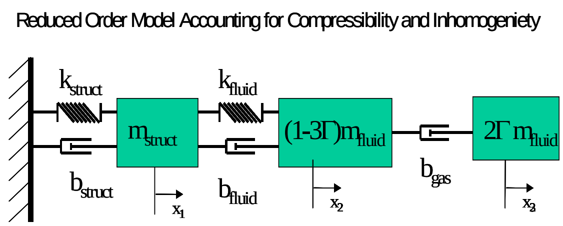

Figure 7: Schematic of Lumped Parameter Model for a Compressible, Inhomogeneous, Aerated Fluid contained in Tube undergoing Harmonic Motion

The equations of motion of the above lumped parameter model, assuming solutions in the form of $e^{s\tau}$ where s is the complex frequency, can be expressed in non-dimensional form as:

$$\begin{bmatrix} s+2\alpha\zeta_f Q+2\zeta_s & 1+\alpha Q^2 & -2\alpha\zeta_f Q & -\alpha Q^2 & 0 & 0 \\ -1 & s & 0 & 0 & 0 & 0 \\ 2\zeta_f Q & -Q^2 & (1-3\Gamma)s+2\zeta_f Q+2\zeta_g & Q^2 & -2\zeta_g & 0 \\ 0 & 0 & -1 & s & 0 & 0 \\ 0 & 0 & -2\zeta_g & 0 & 2\Gamma s+2\zeta_g & 0 \\ 0 & 0 & 0 & 0 & -1 & s \end{bmatrix} \begin{Bmatrix} y_1 \\ x_1 \\ y_2 \\ x_2 \\ y_3 \\ x_3 \end{Bmatrix} = 0$$

The parameters governing the dynamic response of the model are defined in Table 1.

Solving the sixth-order eigenvalue problem described above provides a means to assess the influence of the various parameters on the observed density. The natural frequency of the primary tube mode predicted by the eigenvalue analysis is input into the frequency / density from the quasi-steady, homogeneous model to determine the apparent density of the fluid as follows.

$$\rho_{apparent} = \frac{\rho_{liq}}{\alpha}\left(\frac{f_s^2}{f_{observed}^2} - 1\right)$$

As a baseline condition, a "representative" coriolis meter with parameters given in Table 2, was analyzed.

For a given coriolis meter, the level of aeration has a dominant effect on the difference between actual and apparent mixture density. However, other parameters identified by the lumped parameter model also play important roles. For example, the damping parameter associated with the movement of the gas bubble relative to the fluid within the tube, ζ_{gas}, is an important parameter governing the response of the system to aeration. The influence of ζ_{gas} on the apparent density of the mixture is illustrated in Figure 8. As shown, for ζ_{gas} approaching zero, the apparent density approaches 1-3Γ, i.e., the meter under reports the density of the aerated mixture by 2Γ. However, as the ζ_{gas} is increased, the apparent density approaches the actual fluid density of 1-Γ. The model does asymptote to the identical results of Hemp et al, 2003 due to the compressibility in the model.

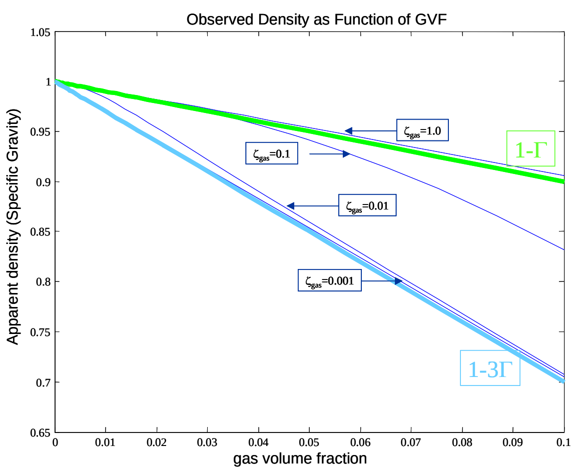

Figure 8: Apparent Density as Function of GVF for Various Levels of Damping Associated with the Movement of the Gas Bubbles within the Fluid

The influence of compressibility is illustrated in Figure 9, in which the model-predicted observed density is shown as function of gas volume fraction for a range of meters differing only in natural frequency of the tubes. As shown, the natural frequency of the tubes, primarily through the influence of the reduced frequency of operation at a given level of aeration can significantly influence the relation between the actual and apparent density of an aerated fluid.

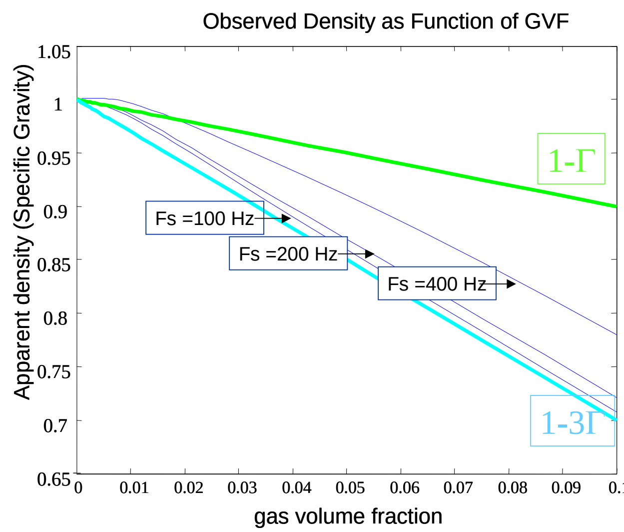

Figure 9: The effect compressibility for the baseline configuration with increasing tube natural frequency.

Speed-of-Sound Augmented Coriolis Meter

Results for the lumped parameter model presented above confirm long recognized accuracy degradation of vibrating tube density meters attributed to aeration. The models can be used to illustrate qualitatively the role of several non-dimensional parameters that govern the performance of the meters in aerated fluids. It can be concluded from these models that gas volume fraction plays a dominant role, with several other parameters including gas damping ζ_{gas} and reduced frequency also influencing performance.

Although simplified models may provide some insight into the influence of various parameters, quantitative models remain elusive due to the inherent complexity of multiphase, unsteady fluid dynamics. Furthermore, the difficulty associated with correcting for the effects aeration in the interpreted density of the liquid is compounded not only by the transformation of the coriolis meter from a well understood device operating in homogeneous, quasi-steady parameter space into a device operating in a complex, non-homogeneous, unsteady operation space, but also by the inability of current coriolis meters to precisely determine the amount of aeration present in the process mixture.

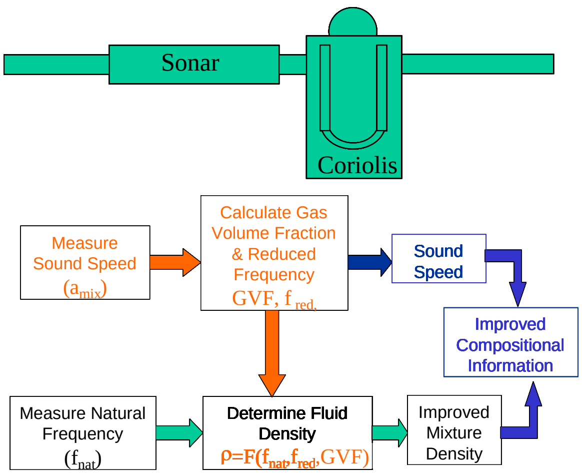

Figure 10: Schematic of the a Speed of Sound Augmented Coriolis Meter

This paper presents an approach in which a speed-of-sound measurement of the process fluid is integrated with the natural frequency measurement of a vibrating tube density meter to form a system with an enhanced ability to operate accurately in aerated fluids. Introducing a real time, speed-of-sound measurement address the effects of aeration on multiple levels with the intent to enable vibrating-tube-based density measurement to continue to report liquid density in the presence of entrained air with accuracy approaching that for a non-aerated liquid. Firstly, by measuring the process sound speed with process pressure, the aeration level of the process fluid can be determined with high accuracy on a real time basis. Secondly, the real time measurements of sound speed, and the derived measurement of gas volume fraction, are then utilized with empirically derived correction factors to improve the interpretation of the measured natural frequency of the vibrating tubes in terms of the density of the aerated fluid. Thirdly, the combined knowledge of aerated mixture density and aerated mixture sound speed, enable the determination of the non-aerated liquid component density, providing improved compositional information. Note liquids phase includes pure liquids, mixtures of liquids, as well as liquid / solids mixtures. A schematic of a speed-of-sound augmented coriolis system is shown in Figure 10.

Experimental Data

A facility was constructed to experimentally evaluate the performance of coriolis meters on aerated water. A schematic of the facility is shown in Figure 11. The facility uses a mag meter operating on single phase water as a reference flow rate and the sonar-based meter to monitor the gas volume fraction of the aerated mixtures.

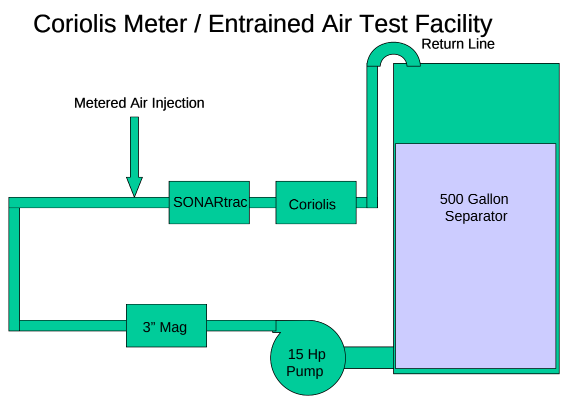

Figure 11: Schematic of the Coriolis Entrained Air Test Facility

The density of the liquid component of the aerated liquid, i.e. the water, was assumed constant. Several coriolis meters of various designs and manufactures were tested. Figure 12 shows apparent density measured by a coriolis meter with 1 inch diameter tubes with a structural resonant frequency of 100Hz. Data were recorded over flow rates ranging from 100-200 gpm and coriolis inlet pressures of 16 to 26 psi. The theoretically correct density of the aerated mixture density factor of $1-\Gamma$ is shown, as is the result from quasi-steady inviscid bubble theory of $1-3\,\Gamma$. Density factor produced by the lumped parameter with the ζ_{gas} tuned to 0.02 is also shown. As shown, the apparent density of the coriolis meter is highly correlated to the gas volume fraction as measured by the sonar-based meter. The lumped parameter model appears to capture the trend as well.

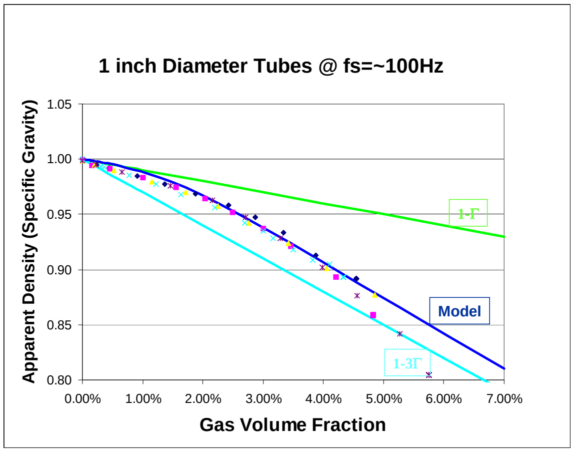

Figure 12: Experimental Data for a Coriolis Meter with 1inch Diameter Tubes with Natural Frequency of 100 Hz. Results from Lumped Parameter Model with ζ_{gas} =0.02 are presented as well.

Figure 13 shows the apparent density measured by the Coriolis meter with 1 inch diameter tubes with a structural resonant frequency of ~300Hz. Data was recorded over a similar range of flow rate and inlet pressures as the previous meter. Again, the theoretically correct density of the aerated mixture density factor of 1-Γ is shown, as is the result from quasi-steady inviscid bubble theory of 1-3 Γ. Density factor produced by the lumped parameter with the ζ_{gas} empirically tuned to 0.007 is also shown. As with the other meter tested, the apparent density of the coriolis meter is highly correlated to the gas volume fraction as measured by the sonar-based meter. The correlation between the output of the lumped parameter model and the output of the density meter is not as good in this case, however, the lumped parameter model still provides a useful framework for assessing the impact of aeration on the apparent density of the process fluid.

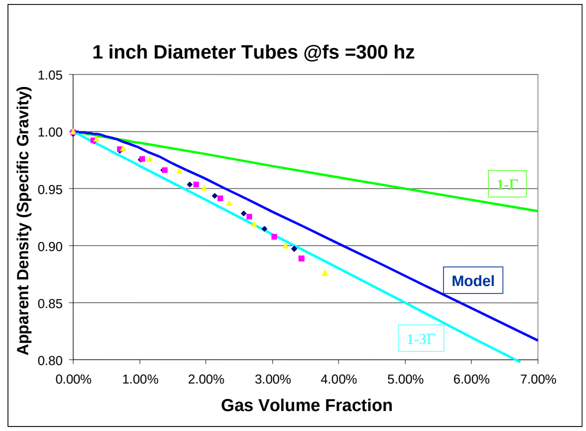

Figure 13: Experimental Data for a Coriolis Meter with 1inch Diameter Tubes with Natural Frequency of 300 Hz. Results from Lumped Parameter Model with ζ_{gas} =0.007 are presented as well.

The performance of a speed-of-sound enhanced coriolis density measurement operating in the presence of entrained air is illustrated in Figure 14. The data shows the time histories of the apparent density, entrained air, and corrected liquid density during an approximately 50 minute period over which the density meter was subjected to varying amounts of entrained air ranging from 0 to 3%. The data presented in Figure 13 was used in conjunction with the real time entrained air measurement to quantify the difference between the actual liquid density and the apparent liquid density during the transient. As shown, the accuracy of the liquid density reported by the speed-of-sound enhanced meter is significantly improved over the apparent density output by the baseline meter.

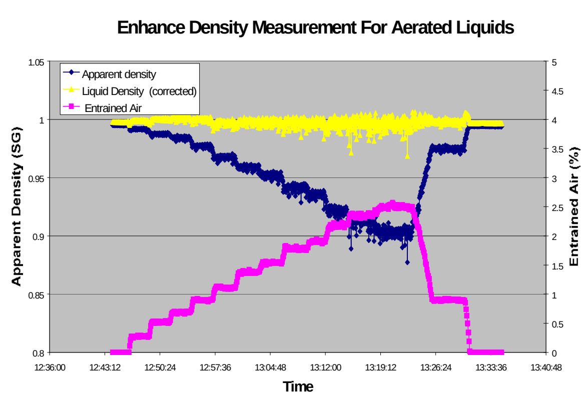

Figure 14: Time History of Performance of Speed-of-Sound Augmented Coriolis Density meter Operating on Aerated Water

Summary

Experimental data and analytical results demonstrate the significant impact that entrained gases have on the accuracy of vibrating tube based density measurement. Analytical models were presented illustrating the how the effects of increased fluid compressibility and inhomogeniety can introduce significant error in the interpreted density of the process fluid. Analytical models illustrated how the impact of aeration is linked to the gas volume fraction of the process fluid, the reduced frequency of the vibrating tubes, and other parameters.

Experimental data was presented demonstrating how the advantages associated with combining a real time measurement of gas volume fraction and reduced frequency with a vibrating tube based density meter can significantly improve the accuracy of both the aerated mixture density measurement as well as the measurement of the non-aerated liquid portion of the mixture.

Acknowledgements

The results presented in this work have benefited from several years of development efforts in flow measurement based on sonar technology. The authors gratefully appreciate the efforts of the many colleagues and co-workers that have contributed to results presented herein.

References

Bisplinghoff, R. L., Ashley, H., and Halfman, Robert, Aeroelasticity, Addison-Wesley Publishing, ISBN 0-486-69189-6, 1955.

Hemp, J. and Sultan, G., Modeling of the Coriolis Mass Flowmeter, Journal of Sound and Vibration, (1989) 132(3), 473-489.

Munjal, M.L., Acoustics of Ducts and Mufflers, John Wiley and Sons, ISBN 0-471-84738-0, 1987

Landau, L.D. and Lifshitz, E.M, Fluid Mechanics 2nd Edition, Course of Theoretical Physics, Volume 6, Section 11, Butterworth-Heinenann, ISBN 0-7506-2767 0, 1987

Hemp, J. Yeung, H. and Kassi, L., Coriolis Meter in Two Phase Conditions, Department of Process and Systems Engineering, Cranfield University, 2003

Hemp, J. and Sultan, G., On the Theory and Performance of Coriolis Mass Flowmeter, Proceedings of the International Conference on Mass Flow Measurement, IBC technical Services, London, 1989.

Gysling, D. L. and Loose, D. H., “Sonar-based Volumetric Flow and Entrained Air Measurement for Pulp and Paper Applications”, presented at the TAPPI Spring Technical Conference, Paper 58-1, Chicago, IL, May, 2003

Symbol	Description	Definition
α	Mass ratio	m_{fluid}/m_{struct}
Q	Natural Frequency Ratio	$\omega_{fluid}/\omega_{struct}$
ζ_f	Critical Damping Ratio of Fluid System	$b_{fluid}/(2m_{fluid}\omega_{fluid})$
ζ_s	Critical Damping Ratio of Structural System	$b_{struc}/(2m_{struct}\omega_{sstruc})$
ζ_g	Critical Damping Ratio of Structural System	$b_{gas}/(2m_{fluid}\omega_{struct})$
τ	Non-dimensional time	$t\ \omega_{struct}$
y	Non-dimensional temporal derivative of x	$dx/d\tau$

Table 1: Definition of Non-dimensional Parameters Governing the Equation of Motion for the Lumped Parameter Model of a Tube Filled with a Compressible, Aerated Fluid

Parameter	Description	Value
f_s	Structural Frequency of Tubes	100 Hz
α	Mass ratio	1.25
ζ_{struct}	Critical Damping Ratio - structure	0.01
ζ_{fluid}	Critical Damping Ratio – fluid	0.01
ζ_{gas}	Critical Damping Ratio – gas	0.01
Q	Frequency Ratio	As determined by sound speed of air / water at STP and structural parameters
D	Tube diameter	1.0 inches

Table 2: Parameters Defining the Baseline Vibrating Tube Density Meter