

Application of Sonar-Based, Clamp-on Flow Meter in Oilsand Processing

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Abstract

Data is presented demonstrating the applicability of sonar-based, clamp-on flow measurement to several, long-standing flow measurement challenges within the oilsands industry, including hydrotransport, coarse tailings, and bitumen froth flow lines.

Sonar-based flow measurement technology was developed and field proven in the oil and gas production industry over the last five years and provides robust, accurate volumetric flow rate measurement for a broad range of process fluids, slurries, pipes sizes and flow conditions. Sonar-based flow metering technology utilizes an array of sensors to listen to, and interpret, unsteady pressure fields within process flow lines. The methodology is implemented using strain-based sensors which clamp-on to existing process piping. Sonar-based flow monitoring systems determine volumetric flow rate by measuring the speed at which self-generated, coherent flow structures convect past the sensor array. Using similar sonar-based array processing techniques as those used for volumetric flow, sonar-based flow meters can also determine entrained air (or any other gas) levels by measuring the speed at which sound propagates within the process flow lines. The speed of sound in the process flow lines provides an accurate and robust, clamp-on method for determining entrained air levels in aerated liquids. Data is presented showing entrained air levels on a 16-inch diameter froth line exiting a steam-driven deaerator.

Introduction

The oilsands processing industry relies extensively on large-diameter-pipe conveyed slurries to transport materials throughout the process of converting oilsands into synthetic crude oil (Gray, 2002). Historically, obtaining accurate and reliable measurements of these slurries has proven technically difficult and economically challenging, due primarily to their abrasive characteristics and difficult to characterize physical properties of solid/liquid mixtures.

This paper addresses the application of sonar-based flow technology to the flow measurement challenges in the oilsands industry, specifically to three, broadly representative classes of slurries commonly encountered in the oil sand processing industry.

- 1) **Hydrotransport** lines which transport and condition the oil sands for bitumen extraction,
- 2) **Tailings** lines, which return the tailings for eventual use in land reclamation

- 3) **Bitumen froth** lines, which transport the bitumen for additional upgrading.

Sonar-based Flow Meters

Sonar-based flow measurement technology was first introduced into the oil and gas industry in 1998 for use in down-hole multiphase flow metering applications (Kragas, 2002), and is currently being applied to a wide range of other industries, including chemical processing industries, mineral processing and pulp and paper industries (Gysling, 2003).

Sonar-based flow measurements utilize an array of sensors, aligned axially along the pipe, to characterize and interpret naturally occurring, unsteady pressure fields within process piping. Although applicable to single phase flows as well, sonar-based flow measurement techniques were specifically developed for multiphase flows.

Figure 1 illustrates the naturally occurring, self-generated, coherent structures present within turbulent process flow of Newtonian fluids. As shown, the time-averaged axial velocity for a turbulent pipe flow velocity profile is a function of radial position, from zero at the wall to a maximum at the centerline of the pipe. The flow near the wall is characterized by steep velocity gradients and transitions to relatively uniform core flow near the center of the pipe. Naturally occurring, self-generating, turbulent eddies are superimposed over the time averaged-velocity profiles. These coherent structures contain fluctuations with magnitudes on the order of 10% percent of the mean flow velocity and are carried along with the mean flow. Experimental investigations have established that eddies generated within turbulent boundary layers remain coherent for several pipe diameters and convect at, or near, the volumetric averaged flow rate in the pipe (Schlichting, 1979). Although this description is based on the empirical and theoretical understanding of Newtonian fluids, this has proved to be a useful framework with which to consider many non-Newtonian flows such as pulp suspensions, froth slurries and others.

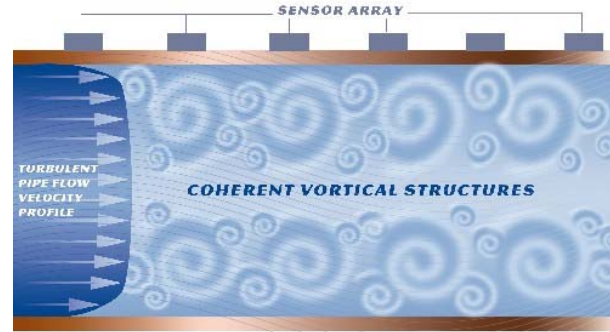


Figure 1: Coherent Structures within Turbulent Pipe Flows

The Convective Ridge

As discussed above, sonar-based flow meters use the convection velocity of coherent structures (eddies) inherent within turbulent pipe flows to determine the volumetric flow rate. The sonar-based algorithms determine the speed of these structures by characterizing both the temporal and spatial frequency characteristics of the flow field. These algorithms have, at their roots, many of the concepts developed to address the beamforming problem in underwater sonar processing (Nielsen, 1991). For a series of coherent eddies convecting past a fixed array of sensors, the temporal and spatial frequency content of pressure fluctuations are related through a dispersion relationship, expressed as follows:

$$\omega = U_{convect} k$$

Here k is the wave number, defined as $k=2\pi/\lambda$ in units of 1/length, ω is the temporal frequency in rad/sec, and $U_{convect}$ is the convection velocity or phase speed of the disturbance, and λ is the spatial wavelength. The dispersion relationship basically states that temporal frequency observed at a fixed location are proportional to the convection speed and inversely proportional to the spatial wavelength of the disturbance.

In sonar array processing, the spatial / temporal frequency content of time stationary sound fields are often displayed using “ k - ω plots”. K - ω plots are three-dimensional power spectra in which the power of a sound field is decomposed into bins corresponding to specific spatial wave numbers and temporal frequencies, the power level is represented by a color map shown to the

right of the $k-\omega$ plot. On a $k-\omega$ plot, the power associated with a pressure field convecting along with the flow is distributed in specific regions that satisfy the dispersion relationship developed above. For turbulent boundary layer flows, this region is termed “the convective ridge” (Beranek, 1992) and the slope of this ridge on a $k-\omega$ plot indicates the speed of the turbulent eddies. Thus, identifying the slope of the convective ridge provides a means to determine the convection speed of the turbulent eddies; and with calibration and knowing the cross-sectional area of the pipe, this slope provides a means to determine the volumetric flow rate.

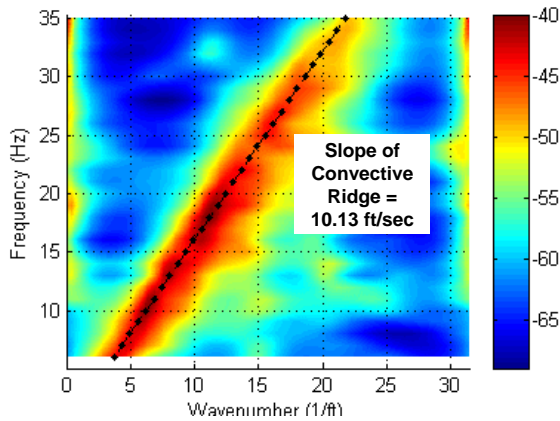


Figure 2: $k-\omega$ plot generated from an array of sensors for water flow in at 10 ft/sec in an 8-inch pipe

Figure 2 shows a $k-\omega$ plot generated from the using the 8-inch diameter sonar-based flow meter, operating at a water-only calibration facility. As shown, the power contours show a well-defined convective ridge. A parametric optimization method was used to determine the “best” line representing the slope of the ridge. For this case, a slope of 10.13 ft/sec was determined.

Calibration

The $k-\omega$ plot shown in Figure 2 illustrates the fundamental principles behind sonar-based convective flow measurements, namely that axial array sensors can be used in conjunction with sonar processing techniques to determine the speed at which naturally occurring turbulent eddies convect within a pipe. The slope of the convective ridge can be calibrated to the

volumetrically averaged flow rate as a low-order function of Reynolds to provides accurate flow measurement. Figure 3 illustrates the how the

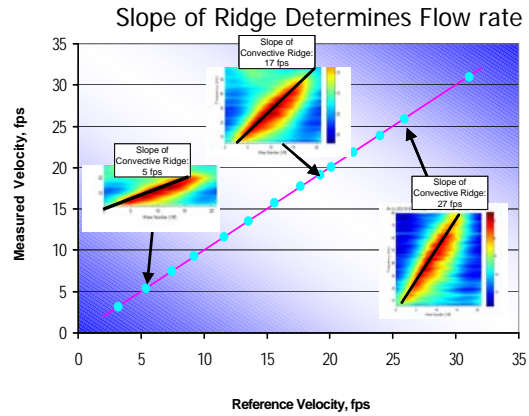


Figure 3: Calibration Data showing representative $k-\omega$ plots at various volumetrically averaged flow velocities

slope of the convective ridge increases with flow rate. Using standard calibration / verification procedures, sonar-based flow meters, with diameters ranging from 3-30 inches, consistently demonstrate accuracies of +/-0.5% accuracy for flow velocities of 3-30 ft/sec. Figure 4 shows data from a family of sonar-based flow meters ranging from 4 to 10 inches in diameter.

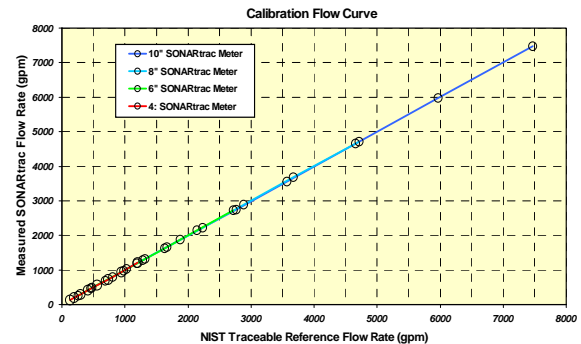


Figure 4: Volumetric Flow Rate measured using Sonar-based convective flow meter versus Reference Flow Rate

Slurries and Non-Newtonian Fluids

Sonar-based flow metering technology has been evaluated in hundreds of industrial applications in numerous industries. Figure 5 shows a comparison of a mag meter and a sonar-based flow meter on 3.5% consistency paper slurry in operating in a commercial paper mill. Although the sonar-based flow meter is reporting flow

~5% lower than the mag meter, apart from this offset, the two measurements are indistinguishable. A representative $k-\omega$ plot constructed from the sonar-based flow meter data shows a clearly defined convective ridge, similar to that observed in the water calibration data. In addition to the convective ridge, a nearly vertical ridge is also present. As will be developed later, the slope of this vertical ridge contains information regarding the speed of sound propagation in the pipe and can be used to determine entrained gas levels within the process mixture.

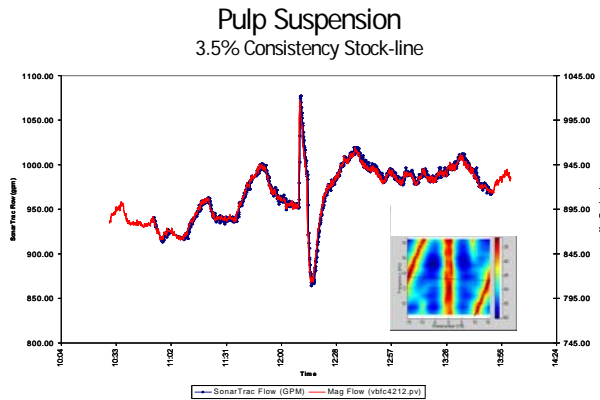


Figure 5: Comparison of a mag meter and a sonar-based flow meter on a 3.5% consistency paper slurry in operating in a commercial paper mill

Oilsands Hydrotransport

Hydrotransport technology is the primary method for conditioning and transporting mined oilsand from the mine to the extraction facilities in which the bitumen is extracted from the water and solids contained in the slurry (Gray, 2002). Hydrotransport lines typically operate at densities of $\sim 1450 \text{ kg/m}^3$ to 1600 kg/m^3 with flow velocities of 4 to 5 m/sec. These flows present several challenges to conventional flow metering technology. Firstly, the flows are extremely abrasive, containing >50% solids by mass with particle distributions ranging from several microns up to several inches in diameter. These flows typically exhibit some level of stratification (Crowe, 1998), as evidenced by the preferential wear of the lower portion of hydrotransport lines, requiring the pipes to be rotated on a periodic basis. The presence of ~10% by volume of non-conductive bitumen

further complicates the flow measurement, as does the possibility of the hydrotransport line containing up to several percent of entrained air by volume.

Currently, modified venturi (or wedge) meters are the predominant devices used to measure flow rates in hydrotransport lines. Mechanical wear of these meters results in high maintenance, calibration, and replacement costs, providing an incentive for operators to evaluate alternative measurement technologies. In efforts to reduce total lifetime costs, eliminate process interruptions due to flow meter maintenance and improve accuracy, sonar-based flow meters have been evaluated on several Oilsand hydrotransport lines. The initial technology evaluation was performed at a pilot facility at Syncrude Research in Edmonton during June 2003.

The pilot facility was designed to simulate the commercial hydrotransport and primary separation processes of oil sands on a reduced scale. A sonar-based flow meter was evaluated on 4-inch line, using a slurry mixture composed to simulate hydrotransport operation. The meter was evaluated as composition of the process

Volumetric Flow & Entrained Measurement

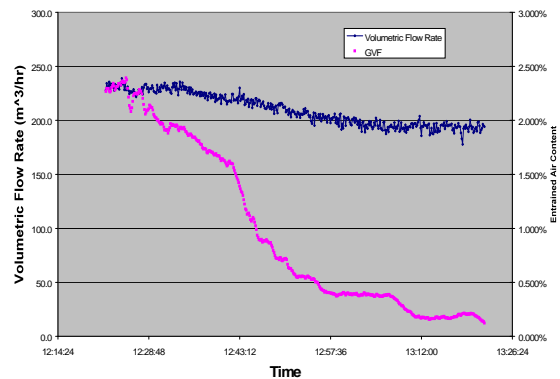


Figure 6: Volumetric Flow and entrained air measurement on 4-inch pilot hydrotransport line

mixture was transitioned from predominately aerated-water mixture to a non-aerated oilsand slurry. The sonar-based flow meter measured total volumetric flow and entrained gas volume fraction. The results of the test are shown in Figure 6. During the ~1 hour test, the entrained air levels are varied. The sonar-based flow meter

indicated that the entrained air transitioned from ~3.5% to less than 0.2%, spanning the levels of entrained air anticipated in commercial hydrotransport lines.

A wedge meter and density meter served as reference measurements. Figure 7 shows that the sonar-based flow meter agrees reasonably well with the reference meter, reporting ~8% lower than the wedge meter for the low-density slurry and reporting ~3% higher than the wedge meter for the high-density slurry.

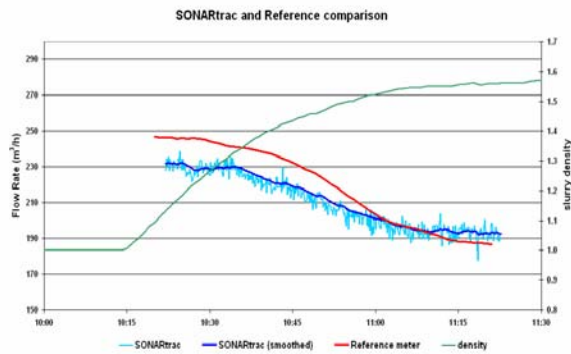


Figure 7: Wedge flow meter comparison with Sonar-based flow meter on 4-inch pilot hydrotransport line

A sonar-based decomposition of the pressure field with the 4 inch slurry lines is shown in Figure 8 for the hydrotransport slurry operating at a relatively low solids content, density ~1.2,

Multiphase Slurry VF & SoS Measurement

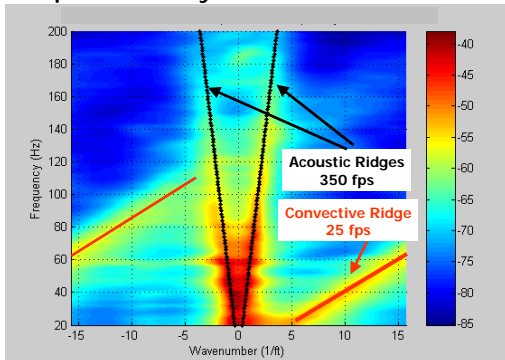


Figure 8: Representative k- ω plot from 4 inch hydrotransport slurry flowing at 21 fps with a density of ~1200 kg/m³ and 3.5% entrained air

but a high aeration level of ~3.5%. The frequency range of the k- ω plot was selected to

show the acoustic and convective ridges simultaneously.

Figure 9 shows a k- ω plot for the hydrotransport operating in full solids loading of ~1500 kg/m³ and relatively low entrained air level of ~0.2%. As shown, the convective ridge is clearly evident and, the comparison of the k- ω plot for hydrotransport slurries (Figure 9) with that for water (Figure 2) is a good indication of applicability of sonar-base flow measurement to oilsand slurries.

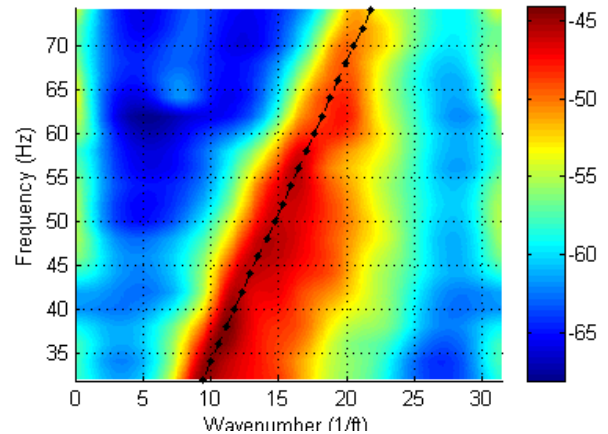


Figure 9: Representative k- ω plot from 4 inch hydrotransport slurry flowing at 21 fps with a density of ~1500 kg/m³



Figure 10: Installation of Sonar-based flow meters on 27inch commercial hydrotransport line

Commercial Hydrotransport Line

Following the demonstration at the pilot facility, a sonar-based flow meter was evaluated on a 27 inch Hydrotransport line at the Syncrude Mildred Lake facility in September 2003. The meter was installed near the discharge of a several-mile-long, hydrotransport line initiating

in the North mine. A picture of the installation is shown in Figure 10. The clamp-on meter was installed ~10 diameters downstream of a ~30 degree elbow, on a ~30 degree incline.

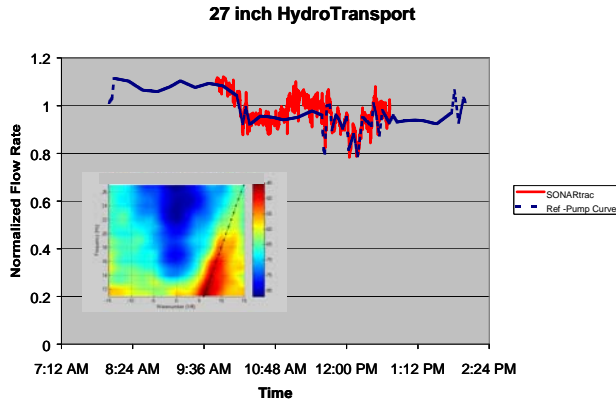


Figure 11: Volumetric flow rate on 27 inch Hydrotransport line- Comparison between Sonar-based flow meter and flow derived from pump-curves

Data recorded from this installation is shown in Figure 11. A reference flow rate, provided by the plant DCS and inferred from pump curves, is also presented. Over the ~ 4 hours of operation, the sonar-based flow meter was in good agreement with the flow inferred from the pump curves, with both flow measurements tracking the same dynamic changes and reporting the same averaged flow rates. It should be noted that, while accuracy is important, the reference measurement available on this line, and most commercial hydrotransport lines, are typically not of sufficient accuracy and tracability to draw definitive conclusions regarding the accuracy of the sonar-based flow meter. The objective was to demonstrate that sonar-based flow measurement principles are applicable to hydrotransport lines and that the sonar-based flow measurement provides a flow measurement consistent with the various other methods currently used to track flow rates in these lines. To this end, the sonar-based, $k-\omega$ decomposition of the pressure field within the hydrotransport is also shown on Figure 11. As shown, the $k-\omega$ plot exhibits a well-defined convective ridge, demonstrating that, like numerous other flows in numerous other applications, oilsand hydrotransport slurries contain self-generated,

coherent structures that can be observed to convect down the pipe at, or near, the volumetrically averaged flow.

Tailings Slurry Line

Tailings slurries transport the water and solids that remain after the bitumen has been extracted from the oilsand mixtures. Tailings from the extraction process are typically pumped into tailings ponds for additional processing and eventual use in land reclamation. Monitoring and optimizing the flow and transport of tailings is an important aspect of bitumen extraction, however, monitoring tailings slurries presents many similar challenges as those seen in hydrotransport lines.

A sonar-based flow meter was evaluated on a 24 inch, commercial tailings lines in September 2003 at Syncrude’s Mildred Lake facility. Figure 12 shows the location of the meter installed under the hoarding. As shown, the sonar-based flow meter was clamped onto the tailings line ~10 diameters downstream of a modified venturi flow meter. The output of the venturi flow meter was provided as a reference flow measurement. Here again, the venturi flow rates serve as a representative measure of the flow rate and, due to issues associated with erosion and the complex nature of solid / liquid flows, contains a significant level of uncertainty regarding absolute flow rate accuracy.

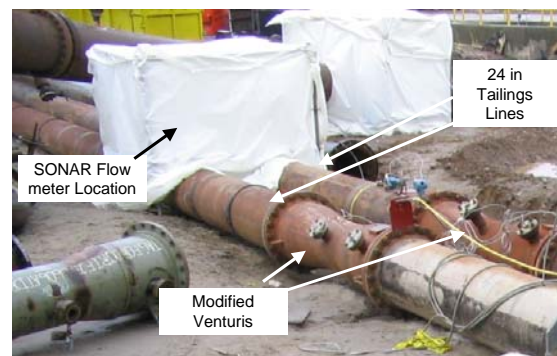


Figure 12: Location Sonar-based flow meter on 24inch commercial tailings line

Results for the evaluation are shown in Figure 13. During this period the tailing lines were operating at a typical solids loading levels, with a mixture density of ~1500 kg/m³. As shown,

the sonar-based flow meter is reporting flow rate ~15% below those reported by the venturi, however, the dynamic behavior of the meters is quite similar. Note the sonar-based flow data has a noticeably faster update rate. A representative sonar-based $k-\omega$ plot is also shown in Figure 13, indicating that, like hydrotransport flow lines, tailings lines generate naturally occurring, coherent disturbances that convect at, or near, the volumetrically averaged velocity in the pipe. As mentioned above, no definitive conclusions regarding accuracy can be drawn from this test.

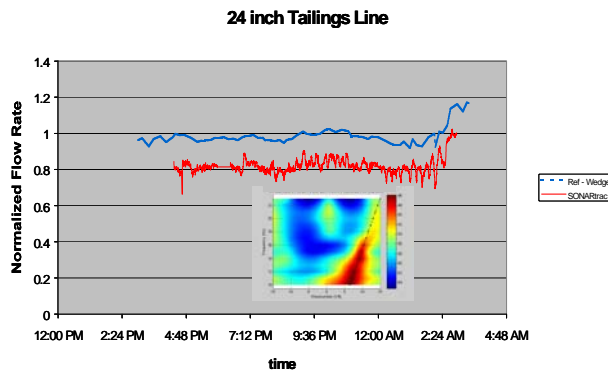


Figure 13: Volumetric flow rate on 24 inch Hydrotransport line- Comparison between Sonar-based flow meter and venturi meter

Bitumen Froth Applications

The end product of the separation processes is bitumen. Bitumen is the primary component in bitumen froth, which typically contains 60% bitumen, 30% water, and 10% fine solids and clays. The transport of bitumen through pipelines has received a significant amount of study in recent years. The challenge in transporting bitumen through pipelines is derived from its high static viscosity, on the order of 10^5 times that of water at standard conditions. If bitumen froth behaved as a Newtonian fluid, i.e. the shear stress was strictly proportional to shear rate, pumping bitumen froth would require several orders of magnitude more power than that required to pump water, rendering pipeline transfer impractical.

Fortunately, bitumen froth is a shear thinning fluid. The viscosity of these types of non-newtonian fluids decreases as the fluid shears. Shear thinning fluids tend to exhibit core-

annular flow regimes, in which the motion of the fluid through the pipe sets up an essentially rigid, core region in the center of the pipe, surrounded by a highly-sheared, less-viscous annular region which effectively lubricates the core flow (Joseph, 1999). From an operational perspective, the primary result of this self-lubrication of bitumen froth (often termed Natural Froth Lubricity or NFL) is a significant reduction (several orders of magnitude) in the pressure drop required to pump a given amount of bitumen froth compared to that predicted by a model using the static viscosity and Newtonian model of the fluid rheology.

Another consequence of the shear-thinning behavior is that the core-annular flow regime establishes undulations in the core/annular interface, often termed “tiger waves” due to the visual patterns observed in laboratory studies using clear wall pipe sections for bitumen froth operating in NFL. These tiger waves have an associated pressure field that is coherent over several pipe diameters and moves with the fluid. Thus, although bitumen froth flows are far from turbulent in the classical sense, they do exhibit a self-generated, coherent pressure field that can be tracked with sonar-based flow meters to determine the volumetric flow.

Sonar-based flow meters were first evaluated on bitumen froth in July 2003 in a test at the Saskatchewan Research Council (SRC), sponsored by Syncrude. The test demonstrated that sonar-based flow methods could be applicable to bitumen froth exhibiting NFL, definite conclusions about the suitability of this technology in a commercial setting could not be drawn due to limitations in the experimental set-up.

Figure 14 shows a convective ridge measured on a 6 inch diameter, schedule 160, pipe for a bitumen froth flowing at ~ 9 fps. As shown, the convective ridge for the bitumen froth is similar to those presented for the other Newtonian and non-newtonian fluids. The sonar-based flow meter reported flow rates in reasonable agreement with available flow estimates used as references. The primary reference available was the rotational speed of the positive

displacement pump, but unfortunately, not enough data points could be collected to draw any definitive, quantitative conclusions regarding the accuracy of the sonar-based flow meter. However, the similarity of the convective ridge identified for the froth flows with those for other applications is a strong indicator of the applicability of sonar-based methods to bitumen froth.

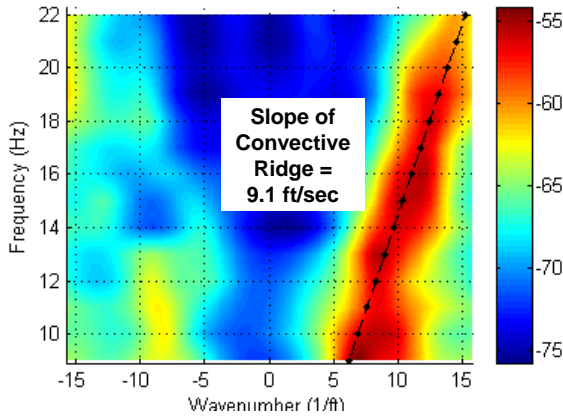


Figure 14: Representative K- ω plot from 6inch, schedule 160 bitumen froth flowing at 9 fps

Following the proof-of-concept testing of sonar-based flow measurement at SRC, sonar-based flow measurement was evaluated on a 16-inch froth line, downstream of froth deaerator at Syncrude’s Mildred Lake facility. In this application, in addition to reporting volumetric flow, the sonar-based flow meter also provided a measure of the entrained air levels of the froth. The meter was installed in a vertical section of pipe with upward flow, approximately 30 ft downstream of a variable speed, centrifugal pump, used to pump the froth from the steam deaerator to a froth header. A picture of the installation is shown in Figure 15. A time history of the output of the sonar-based flow meter is shown in Figure 16. As indicated by motor current, the pump is being cycled on a ~6-8 minute period, resulting in variations in the flow rate and aeration levels of the froth on the same time scale. The flow rate is cycling between ~3000 gpm and ~9000 gpm corresponding to volumetrically averaged flow velocity of ~5 fps to ~15 fps. Again, no quantitative reference was available, however, the time history of the flow rate seems consistent with the time history of the pump motor current.



Figure 15: Installation of Sonar-based flow meter on 16 inch commercial bitumen froth line

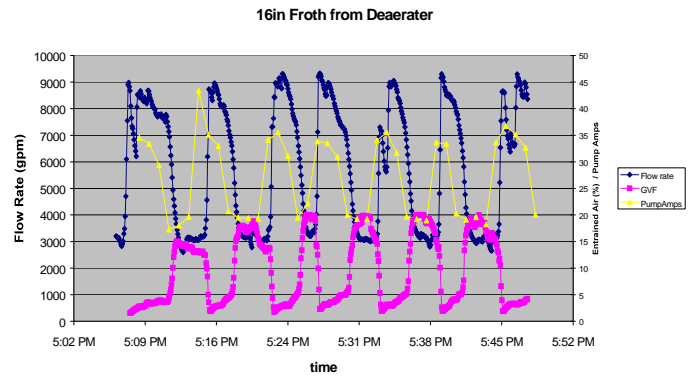


Figure 16: Volumetric flow rate, Entrained Air, and Pump Motor Current for a 16-inch froth line exiting Deaerator

A representative sonar-based $k-\omega$ decomposition of the pressure field within the froth line is shown in Figure 17. The flow rate is at the lower end of the range of flow rates observed at ~6 fps. The convective ridge is clearly evident, as is a nearly vertical ridge in the center of the $k-\omega$ plot. As mentioned for the aerated pulp slurries presented above, this nearly vertical ridge is termed the acoustic ridge. The slope of the acoustic ridge is a measure of the speed at which sound propagates within the process fluid, which, in turn, provides a measure of the entrained air within the process mixture.

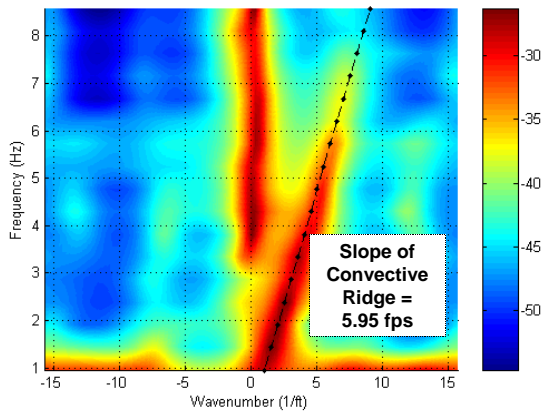


Figure 17: Representative K- ω plot from 16inch, bitumen froth flowing at 6 fps

Figure 18 shows a k- ω plot of the same data series, however, it is looking at a higher frequency range, from ~10-60 Hz, instead of ~1-10Hz in Figure 17. At the higher frequency range, the slope of the acoustic ridge can be precisely determined and used to calculate the amount of aeration in the froth. In this example, sound is propagating at 109 fps, corresponding to ~15% gas volume fraction at a line pressure of 20 psia.

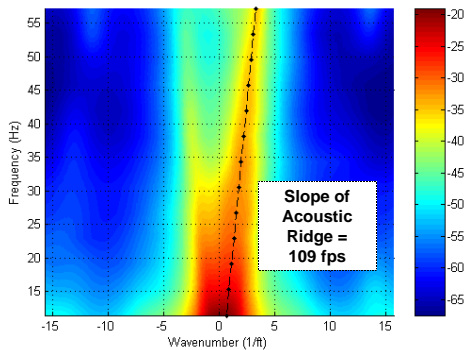


Figure 18: Representative k- ω plot from 16inch, bitumen froth flowing at 6 fps showing Acoustic Ridge indicating a Sound Speed of 109 fps

During the test, no reference for gas volume fraction was available, however, the clearly evident acoustic ridge and the well-established, first principles, link between speed of sound and entrained air levels indicate that the sonar-based technology is well-suited for characterizing the entrained air levels in bitumen froth. Entrained air plays an important role in liberating bitumen from oilsands, enabling the flotation of the bitumen. However, the entrained air impairs

efforts to accurately measure flow rates and density (and therefore percent solids) of bitumen froth streams.

Future Work

The data presented in this paper demonstrates the applicability of sonar-based flow meter to overcome many of the long-standing flow measurement challenges in oilsands processing industry. Long-term commercial trials and installations are currently underway to examine the performance of sonar-based flow meters in several applications including hydrotransport lines, underflow tailings lines, froth lines and final tailings lines. In most cases, the performance of the sonar-based meters is being compared directly to that of mag meters, wedge meters, and venturi meters installed inline, forming a basis to assess the repeatability and reliability of sonar-based flow meters. Additionally, several laboratory tests are planned to assess the accuracy of sonar-based flow meters on slurries over a range of flow regimes typically encountered in oilsand processing, including varying degrees of stratification. The authors intend to report the results of these investigations when the results become available.

Conclusions

Sonar-based flow meters represent a new class of industrial flow meters well-suited to address many of the long-standing flow measurement challenges in the oilsands industry. Laboratory and field data demonstrate the applicability of this clamp-on flow metering technology to three broad classes of slurries common in the oilsand processing industry – Hydrotransport, Tailings, and bitumen froth. Sonar-based flow meter calibrate to within +/- 0.5% on water in calibration labs and the flow rates reported by sonar-based flow meter are, in general, are consistent with available field references. Sonar-based decompositions of the pressure fields, termed k- ω plots, recorded during commercial field trials on these non-Newtonian, inhomogeneous, abrasive, often-aerated slurries are shown to exhibit similar characteristics to those recorded for water and other types of industrial process fluids.

In addition to providing flow rates, sonar-based flow meters provide a real-time measurement of the entrained gases in process lines. It is believed that this device is the first device to monitor entrained air levels on a real-time, full-bore basis and the industry is currently evaluating how best to leverage this new capability.

Acknowledgements:

The results presented in this paper have benefited from several years of development efforts in sonar-based flow measurement technology. The authors gratefully appreciate the efforts of many colleagues and co-workers and that have contributed to results presented herein.

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